Time-Varying Analysis in Risk and Hedge Fund Performance: How Forecast Ability Increases Estimated Alpha *

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Abstract

This paper examines the dynamic of Hedge Funds. Structural change tests indicate a time-varying structure. A linear factor model with time-varying coefficients shows that the proportion of skilled funds is higher than under a static linear factor model. We explain this difference by the time-varying coefficients ability to capture the dynamic part of alphas generated by Hedge Fund managers dynamic strategy. We analyze market exposures during two crises and show heterogeneity within each strategy. Yet, we find that whatever the strategy, exposures are concentrated on the credit spread and the bond risk factors.

Keywords: Hedge Fund performance; time-varying coefficient; nonparametric estimation; kernel methods; multiple structural breaks; multiple hypothesis testing; False Discovery Rate.

JEL Classification: C12, C13, C14, C22, G11, G23

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1 Introduction

Today, investing in Mutual Funds is expected to underperform passive investment strategies on average. As a result, many private as well as institutional investors have turned their attention to Hedge Funds. Hedge Funds are private partnerships using advanced investment strategies, derivatives, leverage and short selling. During alternative investment seminars and conferences, Hedge Fund (HF thereafter) managers pride themselves of being able to produce what they refer to as "absolute alpha"or "absolute return" in the sense that returns are not due to primary asset class performance. Their aim is not to track and try to beat a given stock or bond benchmark, but to focus on pure performance generation. Although we find that performance is due to management decisions based on manager skills, statistical analysis shows that many funds retain significant exposure to different types of market risk factors. It therefore appears essential for investors to determine if these strategies are sensitive to market changes and if they can generate pure alpha thanks to manager skills. This explains the growing attention on HFs performance and their factor exposures.

Mostly, owing to the theory of CAPM or APT, fund performances are assessed using a parametric model with the hypothesis of linearity and constant coefficients. Fung and Hsieh (2001, 2004a) use factors aimed at replicating trend-following strategies. Agarwal and Naik (2000) suggest an using option-based returns approach in order to capture non-linearities. And recently, Bollen and Whaley (2009) study two econometric techniques that focus on risk exposures. Their optimal change point methodology looks for a discrete number of dates in which factor loadings can shift, however, this methodology only accept one single shift in parameters for each fund. An extension of Bollen and Whaley (2009) is provided by Patton and Ramadorai (2011) who show that the variations in leverage cost, the performance of carry trades and of commonly employed benchmarks are important drivers of HFs performance.

Two different approaches have been suggested in order to account for HF managers dynamic allocations. The first approach postulates that the relationship between Hedge Fund returns and market indices are non linear (Agarwal and Naik (2000), Brown et al. (2001), Mitchell and Pulvino (2001)). The second approach considers that betas are not constant during the period studied.

In this paper, we follow the second approach advocated by Bollen and Whaley (2009) and Patton and Ramadorai (2011). While Patton and Ramadorai focus on the behavior of Hedge

Funds at levels that are difficult to scrutinize, we look at the impact on this dynamic by examining the proportion of true alpha as well as the proportions of changes in exposure to different market indexes. Based on a testing approach allowing multiple structural changes, we can address the question whether or not structural changes are due to a crisis. Indeed, if we consider the dynamic behavior of Hedge Fund managers, we should identify at least one structural break revealing a dynamic strategy rather than a passive response to a crisis. We demonstrate these dynamics in all of the observed strategies and find that the relative number of breaks by fund has increased over the last few years. This confirms Chan, Getmansky, Haas and Lo (2005) expectations, according to which, the expected returns of Hedge Funds are likely to be lower and that systematic risk is likely to increase in the future.

Having demonstrated the dynamic behavior of Hedge Fund managers, we introduce an econometric model for Hedge Fund returns which considers this specific point allowing us to relax traditional parametric models and to explore possible hidden structures. For instance, what is the most important manager skill? The skill to pick and choose the right stocks, bonds of any other financial products, or the skill to anticipate market events? If we consider that estimated alpha from a linear model contains these skills, how can we separate and analyze them? Can we find a different proportion of positive-alpha funds?

We here refer to proportion of positive-alpha funds in the sense of Barras, Scaillet and Wermers (2010). They introduce the concept of "proportion of true alphas"according to the False Discovery Rate Approach (FDR hereafter) applied to the world of Mutual Funds and find that only 0.2% of the population of Mutual Funds have generated genuine positive alpha these recent years.

In this paper, we also extend this analysis to the world of Hedge Funds and try to determine globally and by strategy what is the "proportion of true alpha". Furthermore, we apply this approach on the intercept as well as on the betas which, according to our model, can be defined as the market exposures.

To estimate these proportions we rely on a time-varying coefficient model and the FDR approach. We look into the proportion of the fund population that shows a change in market exposures, as well as, the proportion of skilled, unskilled and zero-alpha funds.

We show that the proportion of "skilled"or "unskilled"funds is higher with our model than with a static linear factor model and we explain this difference by the ability of our model to capture the dynamic part of alpha that reflects Hedge Fund manager forecasting ability.

Nevertheless, these results are different depending on the strategy studied. Some strategies like Emerging Market or Event Driven, obtain a percentage of true alphas more or less the same as with the use of our time-varying coefficient or static linear models. Other strategies like Equity Long/Short, CTA or Short Bias reveal a strong difference. Essentially, the majority of Hedge Funds are zero-alpha funds as Barras, Scaillet and Wermers (2010) argued for Mutual Funds. We also find that some strategies obtain a better percentage of true alphas when the market is stressed than when it is stable and vice-versa. This means that like Patton (2009) on market neutral even if a strategy is defined as non-directional, the risk exposure could increase during market turmoil. Therefore after having investigated the performance of Hedge Fund managers, we focus our analysis on the risk behavior. We try to see if a strategy with a common increasing trend to a market exposure during a crisis is possible. And even though each strategy is marked by heterogeneous exposure behavior, we find that the credit spread and the bond risk factors have to be looked into carefully.

The merge between a time-varying coefficient model and the FDR approach defines a new methodology which provides an useful analysis of Hedge Fund selection. Indeed, many articles study Hedge Fund indexes. The use of a multiple hypothesis test analyzes the proportion of skilled funds conditional to the sample study. This method is closer to reality in the sense that portfolio managers often define and look after a peer group. This methodology will determine the percentage of skilled funds conditional to a defined peer group.

The rest of our paper is organized as follows: Section 2 reviews related literature about Hedge Funds modeling and the dynamics in beta[1]. We describe the data in Section 3. Section 4 summarizes the risk factors defined in Fung and Hsieh (2001, 2004). Section 5 tests for structural change by applying the method of Bai and Perron (1998). Section 6 outlines our methodology. Section 7 provides results of our time-varying coefficient model, as well as the application of the FDR to alpha and beta which now offers a new tool for Hedge Fund analysis. Section 8 presents our conclusion. An appendix presents a range of robustness checks of the results.

[1] Readers can find a detailed literature review into the book of Agarwal and Naik (2005).

2 Literature review

Following Fung and Hsieh (1997), many articles have been written on Hedge Funds trading strategies and characteristics by regressing their returns on explanatory factors (Agarwal and Naik (2000), Brown et al. (2001), Mitchell and Pulvino (2001)). Agarwal and Naik (2000) extend this analysis of Hedge Fund performance acknowledging that funds may follow dynamic non-linear trading strategies. Using stepwise regression to identify the independent variables, they find that a put or a call option is the most significant factor for 54% of their funds.

Also, Fung and Hsieh (2002) introduce option strategies into a Sharpe style model and find that, in most cases, these strategies only play a marginal role. One reason the authors give is that they use active and advanced straddle strategies instead of plain-vanilla options. Jrme Detemple, Ren Garcia and Marcel Rindisbacher present a study in which they examine the effects of option-like risk factors on the optimal asset allocation. They are concerned about the portfolio structure and behavior and the impact of timing and selecting abilities. They find that carefully incorporating Hedge Fund classes into asset allocation strategies can be a source of economic gains.

Mitchell and Pulvino (2001) investigate merger-arbitrage strategies and produce useful explicit links between Hedge Fund strategies and observable asset returns. They refer to another important difference between Mutual Funds and Hedge Funds namely that manager investment style changes over time. This problem does not affect Mutual Funds as much as Hedge Funds. Brealey and Kaplanis (2001) present evidence that within each category, Hedge Funds tend to make similar changes to their factor exposures. Similarly, Fung et al. (2006) estimate factor exposures at the time of particular crises. They study vendor-provided fund-of-fund indices, and perform a modified-CUSUM test to find structural break points in fund factor loadings. They note that the break points coincide with extreme market events[2].

The two previous methodologies use a normality hypothesis for independent data hardly verified by Hedge Fund data. See Agarwal and Naik (2001); Amin and Kat (2003); Fung and Hsieh (1999); Lo (2001) for strong evidence of non-normality. Kat and Lu (2002), Brooks and Kat (2002) show that although Hedge Funds offer high mean returns and low standard deviations, returns also exhibit third and fourth moment attributes as well as positive first-order serial

[2]the collapse of Long-Term Capital Management in September 1998, and the peak of the technology bubble in March 2000

correlation[3].

Their distributional characteristics differ depending on the type of Hedge Fund strategies (Anson, 2006).

Recently, Bollen and Whaley (2009) study two econometric techniques that consider changes in risk exposure. They find significant changes in the risk factor parameters in about 40% of their Hedge Funds sample. Patton and Ramadorai (2009) provide an extension to this analysis, according to which, their model outperforms the changepoint regression approach demonstrating that the variations in leverage cost, the performances of carry trade and commonly employed benchmark are important drivers of Hedge Fund risk factors.

3 Database

For this study, we use the Center for International Securities and Derivatives Markets (CISDM) and the HedgeFund.Net databases. The first covers January 1994 to July 2007 and includes dead funds. The full sample contains approximately 9800 funds (Hedge Funds, CTA and Fund of Funds). The second is the largest commercial database of active Hedge Fund, Fund of Fund and CTA products with over 8500 (approximately 3000 Funds of Funds and 5500 Hegde Funds) covering the period from May 1975 to October 2008. The merged database gives roughly 10000 Hedge Funds and 1900 CTAs.

For every fund, we have collected the returns, the strategy and fund type[4] where returns are net of management and performance based fees.

Studying Hedge Funds according to their strategies appears more relevant and a great number of academic literature have dealt with their classification. Fung and Hsieh (1997) and Brown and Goetzmann (2003) identify between five and eight investment styles, whereas Bianchi Drew, Veeraraghavan, and Whelan (2005) only three. Parallel to this, Hedge Fund database providers distinguish between eleven to thirty one investment styles. We therefore follow the twenty three strategies defined by the provider plus the CTA and Fund of Funds. We have grouped the thirty strategies used by HedgeFund.Net to obtain the same twenty three strategies used by the CISDM.

{please insert Table I and II here}

[3]They showed (as Lo & al. (2004)) that monthly Hedge Fund returns may exhibit high levels of autocorrelation.
[4]This database combines four main group, Hedge Funds, Funds of Funds, CTA, and CPO.

4 Factors

Hedge Funds can be divided into four main groups: Market directional, corporate restructuring fund, convergence trading fund and opportunistic funds. A major risk factor can be added to each one: The exposure to the stock market is the major risk affecting market directional funds[5]. The major risk affecting corporate restructuring funds[6] is exposure to the event risk[7]. The same risk affects the convergence trading fund[8]. Furthermore, in every group, each strategy can also have a specific exposure to other risk factors. Risk exposure can therefore drastically change depending on the strategy, which makes defining factors a complex exercise. Fung and Hsieh (2004) show that their seven factor model strongly explains variation in Hedge Fund returns and at the same time avoids multicollinearity[9]. Moreover, they manage to obtain similar results using the Agarwal and Naik (2004) option-based factor model. Their paper includes seven factors and they add an eighth factor on their website[10] that we have also included.

We will therefore follow the eight Hedge Fund risk factors defined in Fung and Hsieh's paper (2004)[11].

These factors are:

Three Trend-Following Factors: Bond, Currency and Commodity[12] which capture a non-linear exposure.

- *Two Equity-oriented Risk Factors*: S&P500 minus risk free rate[13] and Size Spread Factors defined by the Russell 2000 index monthly total return less S&P500 monthly total return.

- *Two Bond-oriented Risk Factors*: Bond Market Factor represented by the monthly change in the 10-year treasury constant maturity yield, and a Credit Spread Factor formed by the monthly change in the Moody's Baa yield less 10-year treasury constant maturity yield.

[5] equity Long/Short, Short selling and equity market timing.

[6] distressed securities, merger arbitrage and event driven.

[7] failure of the proposed transaction.

[8] fixed income arbitrage, convertible bond arbitrage, equity market neutral, statistical arbitrage, and relative value arbitrage.

[9] We use the diagnostic technique presented in chap 3 of Regression Diagnostic by Belsley, Kuh, and Welsh (1980). The diagnostic is capable of determining the number of near linear dependencies in a given data matrix X, and the diagnostic identifies which variable are involved in each linear dependency. We do not detect any multicollinearity with these eight factors.

[10] http://faculty.fuqua.duke.edu/ dah7/.

[11] For more details about the construction of these factors see Fung and Hsieh, 1997, 2001, 2004a.

[12] We thank William Fung and David Hsieh for providing their factors which are downloadable on http://faculty.fuqua.duke.edu/ dah7/DataLibrary/TF-FAC.xls .

[13] 3-month USD LIBOR

- *One Emerging Market Risk Factor*: The MSCI Emerging market minus the risk free rate.

5 Multiple Hypothesis Test

In this short section, we explain why we use the Barras, Scaillet and Wermers FDR approach (2010) reminding of some key elements in their approach.

A first approach is suggested by Kosowski, Naik and Teo (2005). Using a bootstrap procedure, they try to see whether or not Hedge Funds performance can be explained by luck and if it persists at annual horizons. Their methodology tests the skills of a single fund chosen from the universe of alpha-ranked funds. Barras, Scaillet, and Wermers (2010) suggest another approach which provides interesting insights regarding the prevalence of outstanding managers in the whole fund population.

Consider the problem of testing simultaneously M (null) hypotheses, of which M_0 are true. R is the number of hypotheses rejected and is an observable random variable.

	Declared non-significant	Declared significant	Total
True null hypotheses	TNS	TS	M_0
Non-true null hypotheses	NTNS	NTS	$M - M_0$
	m-R	R	

TNS, TS, NTNS and NTS are unobservable random variables. The proportion of errors committed by falsely rejecting null hypotheses can be viewed through the random variable $Q = \frac{TS}{TS+NTS}$. Benjamini and Hochberg (1995) define the FDR Q_e to be the expectation of Q,:

$$Q_e = E[Q] = E\left[\frac{TS}{TS+NTS}\right] = E\left[\frac{TS}{R}\right].$$

It is this quantity that Barras, Scaillet and Wermers (2010) exploit to determine the "true" proportion. Their approach simultaneously estimates the prevalence and location of multiple outperforming funds within a group, examining fund performance from a more general perspective.

We consider several multiple hypothesis testing problems which use this methodology. A first multiple hypothesis testing problem deals with structural change, the second focuses on the proportion of “true alpha”, and the latter on the proportion of change in market exposure.

6 Structural Change

6.1 Testing methodology

This section presents an analysis of structural breaks. If we accept the dynamic behavior of Hedge Fund managers we should detect at least one structural break reflecting a dynamic strategy rather than a passive response to a crisis. For this purpose, we apply the same type of test as Bollen and Whaley (2009) but using Bai and Perron (1998, 2003)'s algorithm[14]. Bollen and Whaley (2009) contribution to this section is to allow several structural breaks tests. We use the test that considers the sup F type test of no structural break ($m = 0$) versus the alternative hypothesis that there are $m = 1, ..., 5$ breaks which is a generalization of the sup F test considered by Andrews (1993) called the Double Maximum Test.

We are concerned by two questions. Bollen and Whaley (2009) find significant changes in the risk factor parameters in about 40% of our sample of Hedge Funds. Can we find the same proportions with this extended test? Do we see a strong difference between strategies? In order to answer to these questions, we build on a multiple hypothesis testing problem by strategy and correct type one error for multiple tests by applying the FDR approach from Barras, Scaillet and Wermer (2010).

{please insert Table III here}

6.2 Empirical Results

The majority of Hedge Funds present structural breaks. By strategy, the minimum percentage of Hedge Funds with breaks is 20% (Distressed Securities) and the maximum is 70% for CTA Systematic. If we consider the most representative strategy (Equity Long/Short, 3519 Hedge Funds), 38% present some structural changes. All these previous results considered track records with more than thirty six months.

[14] We thank Pierre Perron, Jushan Bai and Zhongjun Qu for providing the code on the website: http://people.bu.edu/perron/. This code is a companion to the paper: Estimating and testing structural changes in multivariate regressions (Econometrica, 2007) (developed by Zhongjun Qu).

This confirms that Hedge Fund managers are dynamic in their allocation. But how is their dynamic allocation distributed? Are they grouped around specific events or are they spread over time? Is this dynamic due to a reaction to the market or is it due to the applied strategy? To answer these questions, we follow an approach analyzing the frequency of break dates in the Hedge Fund universe by considering the number of breaks at time t relative to one fund. Since the increase in the amount of Hedge Funds is significant we must take into account in our analysis. We suggest creating a ratio called R_{breaks}, which is defined by the number of breaks at time t divided by the number of funds in activity at time t.

{please insert Graph I,II,III,IV here}

Once again, the results are convincing. Whatever the strategy taken, the structural breaks are spread over time. If we look into these structural breaks during the two observed crises, there are two interesting facts to highlight. In August 1998, the Russian government defaulted on the payment of its outstanding bonds. This default caused a worldwide liquidity crisis with credit spreads widening rapidly throughout the globe. The Russian debt crisis (LTCM) is a crisis which materially affected Hedge Fund returns. This is confirmed by the results from R_{breaks}.

Surprisingly, we notice the opposite for the equity bubble crisis which corroborates Brunnermeier and Nagel conclusions (2002). In 1999, the financial market conditions were very positive, especially for riskier assets; at the same time, a bubble developed. According to Brunnermeier and Nagel (2002), most Hedge Funds, despite irrational levels of valuation, chose to ride the bubble rather than clear their positions. They explain that Hedge Funds heavily tilted their portfolios towards technology stocks without offsetting this long exposure by short or derivatives. They conclude that Hedge Funds deliberately held technology stocks and were able to exploit this opportunity. These arguments are confirmed by the fact that in the majority of these strategies, R_{breaks} is very low, showing fewer structural changes. Thus Hedge Fund managers are dynamic and structural breaks are not necessarily detected during the crisis.

Let us now turn to another problem with the dynamic allocation in relation to the increase in structural breaks. Chan, Getmansky, Haas and Lo (2005) point out that Hedge Funds expected returns are likely to be lower and systematic risk likely to increase in the future. Therefore, if we take their approach into account, Hedge Fund managers should be more dynamic to reach the high-water mark and/or the hurdle rate, and we should detect an increase in our ratio R_{breaks} over the period 2005-2007. Whatever the strategy, we note an increase in the ratio over the 2002-2007 period, but some strategies stand out and show a significant increase. These are the systematic strategies: CTA, Equity Long/Short, Equity Market Neutral, Fixed Income, Global Macro and relative Value Multi Strategies.

Although several papers (Agarwal and Naik (2001); Amin and Kat (2003); Fung and Hsieh (1999); Lo (2001)) show that it is important to take different distributions into considerations, this section has shown that it is insufficient, and that alpha and beta can be dynamic and consequently depend on time. We also show that the risk in Hedge Funds has increased over the last few years essentially due to an increase in dynamic allocation to reach the target returns set by investors. The next section provides a solution to the problem of dynamics in alpha and beta. We suggest a time-varying coefficient model using Fung and Hsieh factors.

7 Time varying exposures

7.1 Factor Model

The previous section shows that the regression coefficients evolve over the observed period. This section presents a model which considers these dynamics as well as certain Hedge funds returns characteristics: Non-normality, limited history (from a couple of months to roughly 150 months), systematic risk captured by a high number of factors.

We recommend using a semi-parametric model relying on the estimation procedure of Fan and Zhang (1999) which allows us to overcome these obstacles. First assumption of this model: Beta is a function of time which can be approximated by a Taylor series. The use of a kernel, whose variable depends on time, relaxes the assumption of normality of Hedge funds returns. It is rare to have a Hedge Fund track record superior to 120 months, this algorithm also allows us to use a number of independent factors whereas the size of the tracks is short. This hypothesis can significantly reduce the modeling bias and avoid the “curse”of dimensionality. Finally, the choice of bandwidth is critical as we use several multiple hypothesis tests which cover a large number of funds. Therefore, we cannot “manually”determine the optimal bandwidth for each fund. We need an estimation procedure that relegates the calculation of the optimal bandwidth to a position of secondary importance. Fan and Zhang (1999) show that in their two-step procedure selects easily the optimal bandwidth.

We now turn to a presentation of the model and its assumptions and explain how we calculate the bandwidth as well as confident interval.

Stone (1977) introduced local linear least squares kernel estimators as a regression estimator which was generalized by Cleveland (1979). Stone (1980, 1982) used local linear least squares kernel and its generalization to higher-order polynomials to show the achievement of his bounds on rates of convergence of estimators of a function m and its derivatives. Fan (1992, 1993) showed in the univariate case that another important advantage of local linear least squares kernel estimators is that the asymptotic bias and variance expressions are particularly interesting and appear to be superior to those of the Nadaraya-Watson or Gasser-Müller kernel estimators. Furthermore, kernel estimators have the advantage of being simple to understand and globally used by researchers; Mathematical analysis and implementation are easy. They are consistent for any smooth m, provided the density of $X_i's$ satisfies certain assumptions.

The time-varying coefficient model assumes the following conditional linear structure:

$$Y_t = \sum_{j=1}^{p} \beta_j(t) X_{jt} + \varepsilon_t = \alpha(t) + X\beta(t) + \varepsilon_t,$$

for a given covariates $(t, X_1, ..., X_p)'$ and variable Y. See appendix for more details on the Time-Varying Coefficient Model (TVCM hereafter).

To conduct statistical inferences such as the construction of confidence interval for $\beta_i(t)$ different methods have been suggested. We opt for the so-called naive bootstrap procedure by Coling and Chiang (2000)[15].

The main advantage of this naive bootstrap procedure is that it does not rely on the asymptotic distributions of $\widetilde{\beta}_i(t)$. Coling and Chiang (2000) recommend another alternative bootstrap procedure suggested by Hoover et al. (1998), which relies on normal critical values approximations[16]. According to the authors, both bootstrap procedures may lead to good approximations of the actual $(1-\alpha)$ confidence intervals when the biases of $\widetilde{\beta}_i(t)$ are negligible[17].

It is well-known in kernel regression that selecting bandwidths is more important than selecting kernel function. In practice, bandwidth may be selected by examining the plots of the

[15] Another paper of Galindo, Kauermann, and Carroll (2000) suggest another bootstrap method based on the wild-bootstrap of Härdle and Marron (1991)

[16] Construct pointwise intervals of the form

$$\widetilde{\beta}_i(t) \pm z_{(1-\alpha/2)} \widetilde{se}^*_B(t),$$

where $\widetilde{se}^*_B(t)$ is the estimated standard error of $\widetilde{\beta}_i(t)$ from the B bootstrap estimators and $z_{(1-\alpha/2)}$ is the $(1-\alpha/2^{th})$ percentile of the standard Gaussian distribution.

[17] They point out that theoretical properties of these bootstrap procedures have not yet been developed.

fitted curves. Nevertheless, in this study we need an automatic bandwidth selection. A great advantage of this two-step estimator is its ability to not be very sensitive to the choice of initial bandwidth. The authors suggest to use cross-validation or generalized cross-validation to determine the bandwidth $\hat{h}$ for the one-step fit. They then use $\hat{\hat{h}} = 0.5\hat{h}$ as the initial bandwidth. Moreover, Colin, Wu and Chiang (2000) suggest applying the "leave-one-subject-out"cross-validation bandwidth[18]

Furthermore, we illustrate our estimator performance by presenting a Monte Carlo study in an appendix. We use the bandwidth defined above and a set of data equal to 50, 100, and 150. The aim of these specific sizes is to respect the average real size of Hedge Fund tracks. Finally, in order to show that our results are independent of our model, we create another time-varying coefficient model which is also based on the work of Fan and Zhang (1999) but using B-spline modeling instead of a local polynomial model. In addition we test the robustness of our methodology by adding another factor (i.e. a liquidity factor[19]) to our previous eight factors. We re-estimate β_i ($i = 1...9$) and compare the results with our first results.

7.2 Methodology

Having introduced the FDR approach and our Time-Varying Coefficient model (TVCM hereafter), we turn to the applications. We look into two areas: Performance of Hedge Funds related to "alpha "and risk exposures.

First, we build a multiple hypothesis test which determines, by strategy, the proportion of "true alpha"during the whole period and during two crisis, i.e., the LTCM and the equity bubble crisis. We are concerned with two aspects. The first focuses on the security selection ability, the second aspect deals with ability to anticipate market events or to handle them (i.e. forecast ability).

Remember that the estimates provided by our TVCM are not a single value over the period studied but a full path.

For the security selection ability, we consider the whole period and we create the t-statistic for each fund by taking the mean of each track as well as the standard deviation[20]. A comparison between the results using two different regression techniques is interesting: We compare our

[18] Appendix I gives a summary to the algorithm.

[19] We thank Lubos Pastor and Stambaugh for providing their liquidity factor (Pastor and Stambaugh (2003)) which is available on the website of Lubos Pastor: http:\\faculty.chicagobooth.edu\lubos.pastor\research.

[20] The t-statistic distributions for individual Hedge Funds are generally non-normal. In order to overcome the non-normality, we use the same approach as Barras, Scaillet, and Wermers (2008), consisting of the use of a bootstrap to more accurately estimate the distribution of t-statistics for each Hedge Funds (and their associated p-values).

TVCM and a static linear factor model with the Newey-West (1987) heteroscedasticity and autocorrelation consistent estimator. We regress the net-of-fee monthly excess return (in excess of the risk-free rate) of a Hedge Fund on the excess returns earned by traditional buy and hold and primitive trend following strategies defined above[21].

For the forecast ability, we look at two notable market events: The LTCM crisis and the Equity Bubble crisis. To properly cover these periods, we have chosen three consecutive months. For the LTCM period we focus on July, August, and September 1998, and for the Equity Bubble period, February, March and April 2000[22]. Using this three month data, we built two t-statistic by crisis which are based on the difference between the second month and the first month and the difference between the last month and the second month. By doing so, we are able to determine whether or not managers are capable of quickly reacting.

Then we look at another multiple hypothesis testing problem which focuses on change in market exposures. We want to analyze the dynamic allocation from Hedge Fund managers. Nevertheless, a slight variation does not necessarily represent a change in their allocations. Therefore we build the t-statistic in the same way as those during the crisis but we test whether the change in exposure is superior to 10% [23]. Thanks to these results, we can look at the proportion of funds that show changes in exposure. It would also be interesting to see what are the highest impacted beta. For that, we calculate the median of the percentage change for each beta.

Thus, this methodology has several advantages for the Hedge Fund analysis process.

First, we are able to analyze the manager skill during a precise short period of time.

Secondly, our FDR approach calculates the percentage of skilled or unskilled funds conditional to the sample study. Third, this method is closer to reality, in the sense that portfolio managers often define and look after a peer group. This methodology will determine the percentage of skilled funds conditional to a defined peer group. Indeed, we do not compare our estimate with an index or a mean performance from a Hedge Fund population nor do we give a specific value for the alpha but we statistically test the percentage of true alpha conditional to our

[21]Kat and Lu (2002), Brooks and Kat (2002) show that the net-of-fees monthly returns of the average individual Hedge Funds exhibit positive first-order serial correlation which is due, according to the authors, to marking-to-market problems. We have removed serial correlation by applying the same methodology as used in Brooks and Kat paper (2001), called the simple Blundell-ward filter; see Geltner (1991, 1993) for an extensive discussion of the motivations for and methodologies to unsmooth returns series. Our appendix contains a brief presentation of their methodology.

[22]End of month of July, August, and September and end of month of February, March, and April.

[23]This methodology is simply a linear relation between two independent variables which, under the condition of normality for $\hat{\beta}_{jt}$ $j = 1, ...N_j$; N_j being the number of funds, assure that the linear relation follows also a normal distribution.

population.

Lastly, we take into account the volatility of estimated alpha. A majority of articles analyzing whether or not Hedge Funds generate alpha, only consider estimated alpha whereas nowadays, financial products analysis consider the performance and the risk factor (for example the volatility). Why should it be different for the manager performance? Within our methodology, we use the ratio estimated alpha and the standard deviation of estimated alpha which is more relevant in assessing the manager performance.

7.3 Empirical Results

Generally speaking, Mutual Fund managers use a buy and hold strategy which means buying a range of financial products following their investment strategy and then holding them according to the time horizon (or investment horizon)[24]. Therefore Mutual Funds are often assimilated to Funds with relative performance. Barras, Scaillet and Wermers (2010) show that only 0.2% of Mutual Funds generate positive alpha and the majority can also be considered as zero-alpha funds. So the question is: Are the results the same for Hedge Funds?

On the one hand, if we estimate a static linear factor model with the Newey-West (1987) heteroscedasticity and autocorrelation consistent estimator, we determine a “static”alpha that does not capture the particularities of Hedge Fund Strategies. In that case, we have found that, whatever the strategies, the majority of Hedge Funds are zero-alpha funds.

On the other hand, when we apply our time-varying coefficient model, we can capture other Hedge Fund manager skills, so that we obtain a non negligible increase of positive alpha funds. We also demonstrate that some strategies obtain a better percentage of positive alpha when the market is stable whereas other strategies obtain a better percentage of positive alpha during market stress. This result means that some non-directional strategies are not really market neutral keeping an exposure to our risk factors especially during market stress. Therefore in a second step, we focus on the risk factors to determine whether or not we find an increase to a specific risk factor.

{please insert Table V here}

We show that the majority of Hedge Funds tend to be marked by an increase in credit spread as well as bond market risk factors during market stress. These results are in line with the Almeida and Garcia (2008) who find that the credit risk factor is the most heavily loaded risk

[24]refer to the time between making an investment and needing the funds.

factor, followed by the bond risk factor. We look into the results for seven out of the twenty four strategies in the next part of this paper.[25]

Results for the CTA

We find a strong difference in estimated alpha between the static factors model (SFM hereafter) and our time-varying coefficient model. The SFM gives a very small percentage of positive and negative alpha funds; 2% and 1% respectively while the TVCM gives roughly 19 and 28%. Although the CTA category handles the two crises, it obtains a strong percentage of positive alpha funds during the Equity Bubble Crisis, ie approximately 27%.

During the Summer of 1998, CTAs experienced one of their best performances[26] while all other Hedge Fund strategies were struggling.

Unsurprisingly, during the two events, the CTAs tend to show a slight increase in the credit spread and the emerging market risk factors equal to 2%. Nevertheless, this sensitivity only affects a small percentage of our population. The majority of CTAs have a relatively stable exposure. We note that approximately 9% of funds show an increase in liquidity during LTCM whereas during the Equity bubble, the CTA strategy keeps the same liquidity.

{please insert Graph XI and table VI here}

Emerging Markets

The emerging market strategy shows a good proportion of stock-picker skilled funds where approximately 12% generate positive-alpha which reveals that the majority of managers are fundamental bottom-up stock-pickers. We get the same results using the SFM which shows that the estimated alpha is more "static"than with other strategies. The proportion of dynamic skilled funds is very positive during both crisis with a strong 40% of positive alpha funds. These results confirm that emerging market equity hedge fund managers saw as real opportunity on the high emerging markets volatility.

During the Equity Bubble and LTCM we notice a stronger dynamic strategy than previously seen where approximately 25 and 55% of our population show an increase in two risk factors: The credit spread and the bond market risk factors. The credit spread risk factor is the most sensitive factor during LTCM, whereas, four out of the eight factors show a sensitivity during

[25]The results and the graphs for the seventeen remaining strategies are available upon request.

[26]Approximately 10 percent in August and 7.5 percent in September according to CSFB/Tremont Managed Futures

the Equity Bubble crisis. LTCM is a crisis that decreases the liquidity of this strategy whereas the equity bubble crisis does the opposite.

{please insert Graph V and table VI here}

Equity Long/Short

Equity Long/Short obtains approximately the same results as the CTA apart from the Equity Bubble crisis. It receives a better percentage with 24% using the TVCM. The SFM give a small 4% and 3% of negative alpha funds. Certain Equity Long/Short specialize in a specific sector like technology, and, unsurprisingly, the forecast ability had a greater impact during the Equity Bubble than during LTCM. Generally speaking, the proportion stays relatively consistent proving their ability to switch from the short to the long position and vice versa.
A small percentage of the population shows an increase or a decrease in exposure during the two crises. Still, we notice a sensitivity to the credit spread risk factor and to the commodities factors during LTCM whereas emerging risk factor is the most sensitive during the Equity Bubble crisis. This strategy stays robust to the liquidity factor whatever the crisis.

{please insert Graph VI and table VI here}

Equity Market Neutral

This strategy gives a very interesting result where the proportion of stock-picker skilled funds is 3% higher than the estimated proportion using the SFM. Therefore the two results are relatively closer than with the other strategies. Furthermore, this result is confirmed by the obtained percentage of true alpha during LTCM with 0%. It does not cope as well during the Equity Bubble crisis. We notice a very small 4% during the first period which reaches a strong 16% in the second period. These percentages corroborate Patton (2009) results where he raised the question about market neutral strategy really being market neutral. Nevertheless we note that like Equity Long/Short, this strategy keeps the same liquidity whatever the crisis studied.

{please insert Graph VII and table VI here}

In relation to the previous percentage of true alpha obtained, it appears essential to analyze the change in exposure. We find that this strategy is the most robust. Yet, for a minority of the population we notice that the credit spread and the commodity show the biggest sensitivity during LTCM and the emerging market factor during the Equity Bubble.

Event Driven Multi Strategy

Event Driven Multi Strategy obtains the worst percentage of skilled-funds with 0% for the SFM and a small 2.5% for the TVCM. The result is confirmed during the two crises with 0% during LTCM. This is not surprising as both crises create several opportunities[27] which are not captured in the period studied. For example, a flood of corporate bankruptcies emerged during the dot-com bust in 2001-2002. On the other hand, this is the strategy which obtains the smallest proportion of unskilled funds. This point confirms the convergence of Hedge Funds and Private Equity[28]. The adaptability of these two managers categories allowed them to survive the changing market conditions and prosper along with their investors.

{please insert Graph VIII and table VI here}

Another point which confirms this convergence, relates to the percentage of the population showing a variation in factor exposure. A small part shows instability during the crisis. This validates the convergence in the sense that distressed debt managers began to pursue longer-term investments as private equity funds. Therefore the only factors which show an increase are the credit spread, the emerging market factor and the commodity factor for LTCM and the emerging risk factor for the Equity Bubble. Surprisingly, because of this convergence, we expected to see a change in liquidity factor. The results however only show a slight variation during LTCM.

Global Macro

Global Macro shows a proportion of stock-picker skilled funds equal to 5% using the TVCM and 1% using the SFM. It obtains one of the biggest groups of unskilled funds with roughly 28%. Unsurprisingly, the percentage of positive alpha during LTCM increases up to 26%. These results confirm that global macro managers have the most extensive investment universe and that they are able to find opportunities. The Equity Bubble crisis also gives a good percentage of positive alpha funds with 18%.

Less than 10% of our population shows an increase or decrease in our general exposure. Credit Spread risk factor stays the most sensitive during LTCM whereas, the size spread factor, the emerging market risk factor and the credit spread risk factor (slightly) are marked by a change in exposure. Less than 5% of our population show a decrease or increase in the liquidity factor.

[27] Invests in mergers, spin-offs, reorganizations, and other announced events.

[28] see Gonzales-Heres and Beinkampen (2006)

We can say therefore that this strategy relatively keeps the same liquidity.

{please insert Graph IX and table VI here}

Short Bias

The SFM give 5% of skilled funds whereas the TVCM 35%. Furthermore, this strong percentage is less than the 71% of positive alpha funds found during LTCM. The Equity Bubble crisis obtains a small 27%: This impressive result confirms the strong dynamic within the strategy. Moreover, the short bias produces the best percentage of variation exposure in. More than 60% of our population shows an increase in different market risk factors. Credit spread is still present during LTCM while Size-spread risk factor has a non negligible sensitivity to bond, commodity and emerging market risk factors during the Equity Bubble. This strategy is the most interesting regarding the liquidity factor. The results show that when the market is stressed, a strong percentage has an increase in liquidity which confirms that it is a very good strategy during market turmoil.

{please insert Graph X and table VI here}

It is important to note that the number of Hedge Funds following this strategy is relatively small, so the result, in our opinion, could be debatable.

Subprime Mortgage Crisis

We begin our empirical analysis by estimating the Hedge Fund manager performances (alpha) using our Time-Varying Coefficient model over the period March 2007 to March 2010. Table VII shows estimated proportions of unskilled and skilled funds by strategy (π_0, π_A^-, π_A^+), as defined in sections 5 and 7.2.

{please insert Table VII here}

We obtain a completely different result in comparison with the other crises studied as the proportion of negative, zero and positive alpha funds are roughly the same. Generally speaking, 1/2 of the hedge funds of each strategy are negative alpha funds, 1/4 of the hedge funds are zero alpha funds and the remaining 1/4 are positive alpha funds. Among all strategies, we estimate that the majority - 75% - are therefore zero or negative alpha funds. 23% are zero-alpha funds which is a strong decrease in comparison with the others crises. The Subprime Mortgage Crisis has strongly increased the proportion of negative alpha funds but surprisingly, we also note a substantial increase of positive alpha funds: between 12.4% for Equity Long/Short to 34.4% for Short Bias. We explain this increase of negative alpha funds by the fact that many Hedge Funds operated with too little capital and used short-term financing to fund the subprime mortgages. When they could not sell these mortgages, many of them were forced out of business. Hedge Funds which did not have an exposure to subprime mortgages find some very interesting opportunities which explains the increase of positive alpha funds. What is the difference with the other crises? During the previous ones, there were always some strategies which performed better than others. For example emerging market had better results during the two previous studied turmoils than globally. The CTAs stayed relatively stable during and after did particularly well during the equity Bubble crisis. Global Macro had a better result during LTCM, etc. In the opposite, all strategies during the subprime mortgage crisis were impacted. We see a switch from the proportion of zero alpha fund to the proportion of negative alpha fund. Meaning that, whatever the strategy, a strong proportion of HF managers which absorb their absolute performance by their management and performance fees became funds that create subperformance.

8 Conclusion

Hedge Funds cover a wide array of strategies with radical differences in terms of risk. Similarities however do exist. Hedge Fund managers strive to focus on positive returns (independent of market conditions), the use of leverage and their structural fees. The characteristics of Hedge fund returns require an econometric model, which ignore the ad hoc error distribution assumption, and focus on the dynamics in beta or a non linearity exposure to the market.

Dynamics in beta is the first subject that this paper addresses. We show that the majority of Hedge Funds have a minimum of one structural break and also underline the accelerated frequency of breaks over the last few years.

To overcome these obstacles, we opt for a time-varying coefficient model and include full set of factors defined in Fung and Hsieh (2001, 2004). The model allows us to define alphas and betas as functions which depend on time and to avoid a parametric assumption. It also covers one of the best overall risk factors as we base our model on the factors of Fung and Hsieh(2004).

In addition, the merge between our time-varying coefficient model and the FDR approach provides a new methodology for Hedge Fund analysis.

First, this model allows us to separate manager skills into two components illustrated by the (stock or bond or funds)-picking and the ability to anticipate market events. It also allows us to see what changes there are in beta exposure or in the manager reactions to the changing market conditions.

Secondly, the FDR approach allows us to evaluate the proportion of skilled fund conditional to the sample study and gets ride of the different biases inherent to Hedge Fund databases. We determine the proportion of skilled funds by looking into the estimated alpha as well as its volatility.

While Barras, Scaillet and Wermers (2010) show us that only 0.2% of Mutual Funds generated positive alpha and therefore the majority can be considered as zero-alpha funds, the different result for Hedge Funds is different. Hedge Fund managers seek absolute returns and try to outperform the market whatever the market conditions. We show that a static factor model, where the results would make Hedge Funds zero-alpha funds, fails to capture this dynamic. In contrast, our model finds a higher proportion of positive alpha funds but also a higher proportion of negative alpha funds. For positive alpha funds, the minimum percentage we find is 2.5% for event driven multi strategy and the maximum: 18.5% for CTA and Equity

Long/Short strategies. As for negative alpha funds, the minimum percentage is 0% for Event Driven Multi Strategy, and maximum, 46% for the Emerging Market strategy. Some strategies also stand out as their percentage of true alpha is higher when the market is stressed than when it is stable and vice versa. This means that even if a strategy is defined non-directional, the risk exposure can increase during market turmoil. Our methodology has also the advantage that it can analyze the changes in risk factors. Looking at each strategy, we can determine the percentage change for our eight factors and evaluate the persistence of betas parameters. Our results show that for all Hedge Funds, two exposures stand out in the down-state of the market. These are the credit spread and the bond risk factors. Finally, the changes in factorial exposure and the proportion of funds give us a solid tool for risk managers and particularly for stress-testing.

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Table I: Number of Hedge Funds by Strategy

See Appendix I for definitions of fund types. The funds used at minimum cover the LTCM and bubble period which represent a track record of a minimum of 36 months. Certain of the funds used are considered as dead funds i.e. they stopped their activities. The database marked by a asterisk give the number of funds covering the 2 specific periods i.e, LTCM period (July, Auguste, and September 1998) and the Equity Bubble years (February, March, and April 2000)

Strategies	**Number of Hedge Funds**			
	CISDM	merge	CISDM*	merge*
Equity Long/Short	2141	3519	1561	2525
Multi Strategy	251	734	142	462
Emerging Markets	445	635	331	461
Sector	387	630	296	468
Equity Market Neutral	322	698	227	519
Event Driven Multi Strategy	224	384	168	293
Global Macro	303	534	205	377
Equity Long Only	148	276	99	174
Single Strategy	114	112	50	50
Fixed Income	153	326	109	260
Distressed Securities	162	268	131	232
Fixed Income Arbitrage	190	314	143	237
Convertible Arbitrage	207	271	181	227
Relative Value Multi Strategy	89	179	74	137
Fixed Income - MBS	74	98	59	69
Option Arbitrage	29	126	17	76
Merger Arbitrage	126	139	111	122
Other relative Value	16	32	7	17
Short bias	51	74	32	56
Regulation D	16	56	13	44
Capital Structure Arbitrage	21	30	13	21
Market Timing	2	3	1	1
Unclassified	58	521	37	369
CTAs (systematic)	1003		759	
CTAs (discretionary)	283	1915	202	1394
FoHFs (multi strategy)	1837		1390	

Table II: Merge between the strategies from the CISDM and HedgeFund.net

See Appendix I for definitions of fund types. This table shows how we have grouped the 31 strategies from HedgeFund.Net and the 23 strategies from CISDM .

CISDM database 23 Strategies	**HedgeFund.Net database** 31 Strategies
Multi Strategy	Multi Strategy Statistical Arbitrage
Equity Long/Short	Equity Long/Short
Short bias	Short bias
Event Driven Multi Strategy	Event Driven
Emerging Markets	Emerging Markets
Merger Arbitrage	Merger (risk) Arbitrage
Fixed Income	Fixed Income (non arbitrage)
Equity Market Neutral	Market Neutral Equity
Global Macro	Macro
Relative Value Multi Strategy	Value
Sector	Small/Micro Cap Finance Sector Technology Sector Energy Sector Healthare Sector
Equity Long Only	Long Only
Distressed Securities	Distressed
Single Strategy	FoF Market Neutral
Unclassified	Asset Based Lending country specific Special situations short-term trading
Fixed Income - MBS	Mortgage
Convertible Arbitrage	Convertible Arbitrage
Fixed Income Arbitrage	Fixed Income Arbitrage
Other relative Value	Other Arbitrage
Market Timing	Market Timer
Option Arbitrage	Option Strategies
Regulation D	Regulation D
Capital Structure Arbitrage	Capital Structure Arbitrage

Table III: Test of Multiple Structural Changes

See Appendix I for definitions of fund types. The funds used at minimum cover the LTCM and bubble period which represent a track record a minimum of 36 months. Listed is a test provided by Bai and Perron (1998) to analyze whether Hedge Funds have some structural. The test considers tests of no structural break against an unknown number of breaks given some upper bound (m = 5). The UDmax and the WDmax differ by their weight methodology, See Bai and Perron (1998) for a full explanation on the different weights.

Strategy	**Double Maximum test**	
	UDmax	WDmax
Equity Long/Short	38%	38%
Multi Strategy	46%	46%
Emerging Markets	34%	34%
Sector	41%	41%
Equity Market Neutral	49%	49%
Event Driven Multi Strategy	35%	35%
Global Macro	42%	42%
Equity Long Only	53%	53%
Single Strategy	71%	71%
Fixed Income	40%	40%
Distressed Securities	20%	20%
Fixed Income Arbitrage	41%	41%
Convertible Arbitrage	41%	41%
Relative Value Multi Strategy	30%	30%
Fixed Income - MBS	58%	58%
Option Arbitrage	57%	57%
Merger Arbitrage	27%	27%
Other relative Value	75%	75%
Short bias	51%	51%
Regulation D	29%	29%
Capital Structure Arbitrage	53%	53%
Market Timing	100%	100%
Unclassified	38%	38%
Fund of Hedge Funds	65%	66%
CTA Systematic	70%	70%
CTA Discretionary	66%	66%

Table V: Estimated proportions of zero-alpha, unskilled, and skilled funds from merge database

See Appendix I for definitions of fund types. This table displays the estimated proportions of zero-alpha, unskilled, and skilled funds for each strategy after applying the False Discovery Rate methodology developed by Barras, Scaillet and Wermers (2010). These results come from the merge between the CISDM and the HegdeFund.net databases. The funds used at minimum cover the LTCM and equity bubble periods which represent a track record with minimum of 36 months. We estimate alphas with the time-varying coefficient model defined in Section 6. We compute the average of estimated alphas which represents the stock-picker ability and the indicators defined in Section 7 during the 2 crises:α_{L1} and α_{L2} for LTCM and α_{B1} and α_{B2} for the Equity Bubble which represents the different market timer abilities. We also gave the result using a linear factor model: the Newey-West (1987) heteroscedasticity and autocorrelation consistent estimator. We do not give some results for Single Strategy because of too few data. Furthermore the results about the other strategies are available upon request.

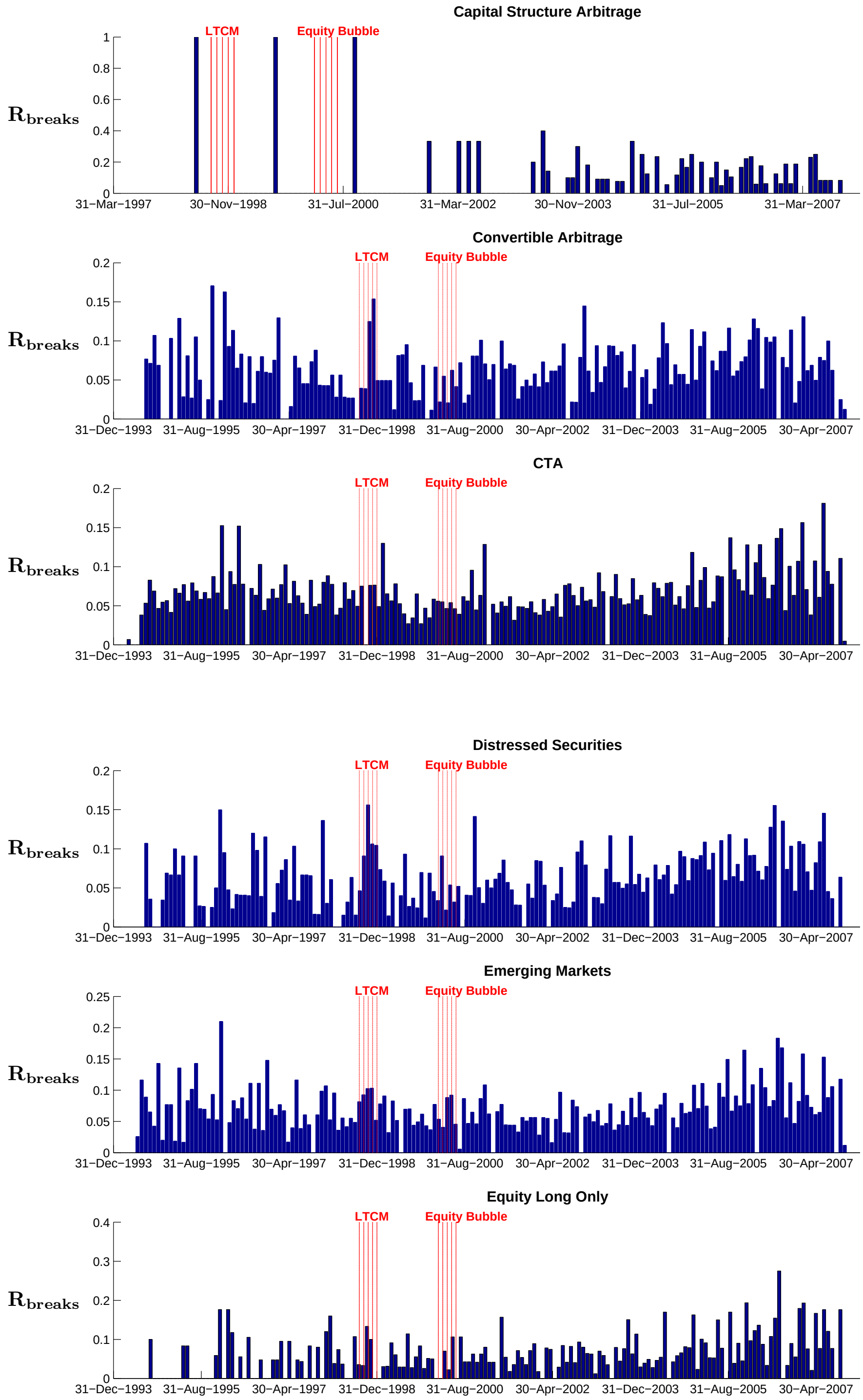
Strategy	Model		α			α_{L1}			α_{L2}			α_{B1}			α_{B2}	
	used	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-
Eq. Long/Short	TVCM	18.6%	58.8%	22.6%	16.0%	61.9%	22.1%	22.8%	62.2%	15.0%	17.0%	58.8%	24.2%	17.0%	58.8%	24.2%
	Nwest	3.8%	93.5%	2.7%												
Emerging Markets	TVCM	12.6%	41.5%	45.9%	44.2%	42.6%	13.2%	38.9%	43.8%	17.3%	44.4%	39.2%	16.4%	43.6%	39.2%	17.3%
	Nwest	12.3%	87.5%	0.2%												
Eq. Market Neutral	TVCM	11.6%	74.1%	14.2%	0.0%	90.4%	9.6%	0.0%	92.2%	7.8%	4.3%	83.2%	12.5%	16.4%	83.2%	0.5%
	Nwest	8.6%	89%	2.4%												
Event Driven M. S.	TVCM	2.5%	97.5%	0.0%	0.0%	97.5%	2.5%	0.0%	99.8%	0.2%	0.9%	90.7%	8.4%	0.9%	90.7%	8.4%
	Nwest	0%	100%	0%												
Global Macro	TVCM	5.2%	67.0%	27.8%	26.1%	69.1%	4.8%	26.2%	66.8%	7.0%	18.5%	59.9%	21.6%	18.5%	59.9%	21.6%
	Nwest	1%	96%	3%												
Short bias	TVCM	35.2%	9.5%	55.2%	71.4%	0.0%	28.6%	71.4%	0.0%	28.6%	49.0%	19.0%	31.9%	49.0%	19.0%	31.9%
	Nwest	5.3%	63.6%	31.2%												
CTA	TVCM	18.8%	53.1%	28.1%	17.0%	60.2%	22.8%	17.0%	60.2%	22.8%	27.2%	55.3%	17.5%	27.2%	55.3%	17.5%
	Nwest	2%	97%	1%												

Table VI: Liquidity factor: estimated proportions of funds that show a change in liquidity during the two crisis

See Appendix I for definitions of fund types. This table displays the estimated proportions of funds that show a change in liquidity for each strategy after applying the False Discovery Rate methodology developed by Barras, Scaillet and Wermers (2010). These results come from the merge between the CISDM and the HegdeFund.net databases. The funds used at minimum cover the LTCM and equity bubble period which represent a track record with minimum of 36 months. We use the liquidity factor provided by Pastor and Stambaugh (2003) which is available on the website of Lubos Pastor: http:\\faculty.chicagobooth.edu\lubos.pastor\research. We estimate the liquidity factor with the time-varying coefficient model defined in Section 6. We compute the average of estimated alphas which represents the stock-picker ability and the indicators defined in Section 7 during the 2 crises:α_{L1} and α_{L2} for LTCM and α_{B1} and α_{B2} for the Equity Bubble which represents the different market timer abilities.

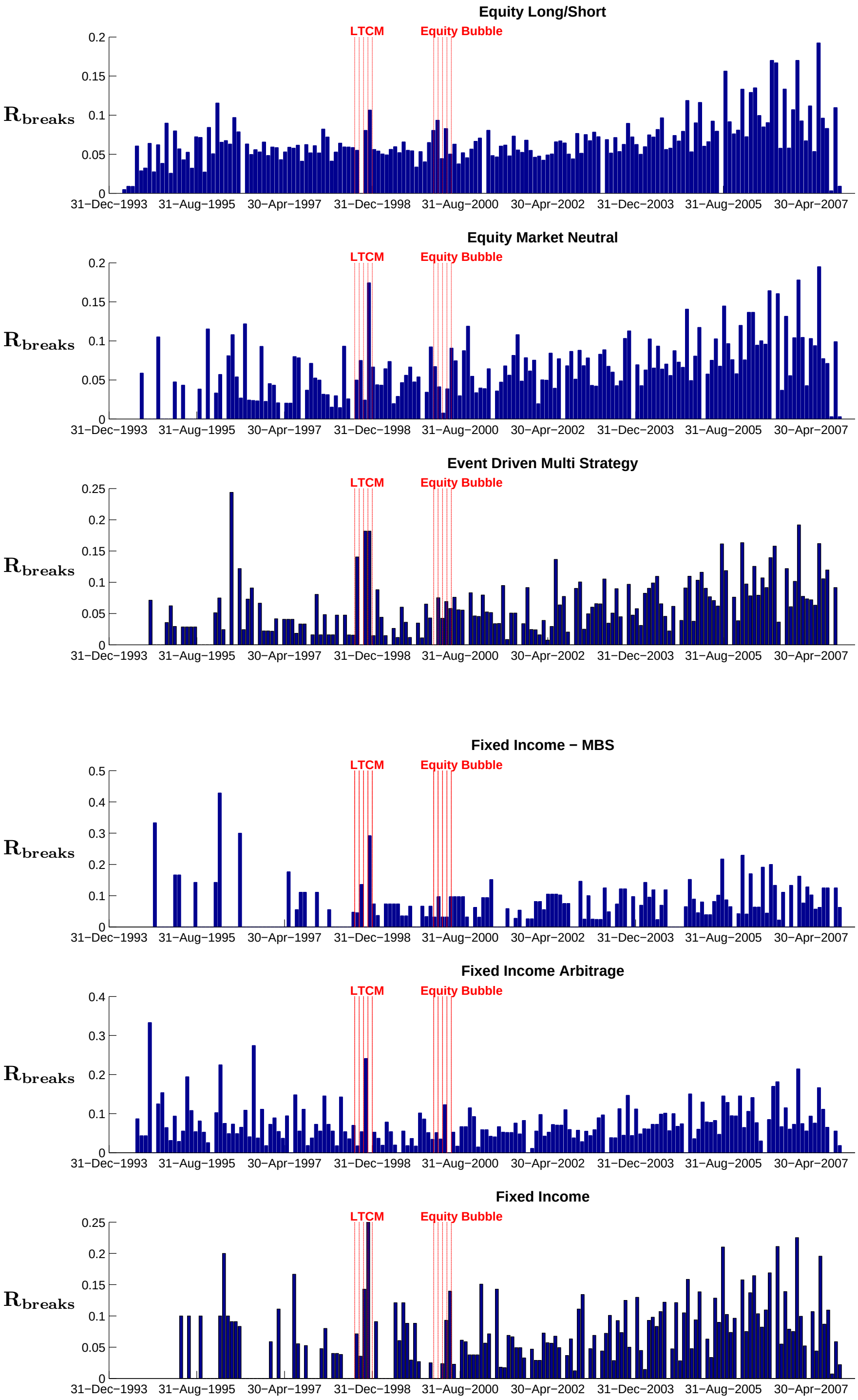
Strategy	Model		Liq_{L1}			Liq_{L2}			Liq_{B1}			Liq_{B2}	
	used	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-	π_A^+	π_0	π_A^-
Eq. Long/Short	TVCM	0.0%	100%	0%	0.0%	100%	0%	1.9%	95.8%	2.3%	1.9%	95.8%	2.3%
Emerging Markets	TVCM	9.8%	86.4%	3.8%	11.1%	87.6%	1.3%	10.1%	81.8%	8.1%	10.1%	81.8%	8.1%
Eq. Market Neutral	TVCM	0.5%	99.5%	0.0%	0.5%	99.5%	0.0%	0.0%	100%	0%	0.0%	100%	0%
Event Driven M. S.	TVCM	6.9%	88.4%	4.7%	6.9%	88.4%	4.7%	0%	100%	0%	0%	100%	0%
Global Macro	TVCM	4.8%	89.9%	5.3%	5.4%	89.9%	4.7%	0.0%	96.8%	3.2%	0.0%	96.8%	3.2%
Short bias	TVCM	13.8%	76.2%	10.0%	23.6%	66.7%	9.8%	23.8%	76.2%	0.0%	23.8%	76.2%	0.0%
CTA	TVCM	8.4%	88.0%	3.6%	9.4%	87.0%	3.7%	0.0%	99.4%	0.6%	0.0%	99.4%	0.6%

Figure I: Structural breaks over the period January 1994 - April 2007



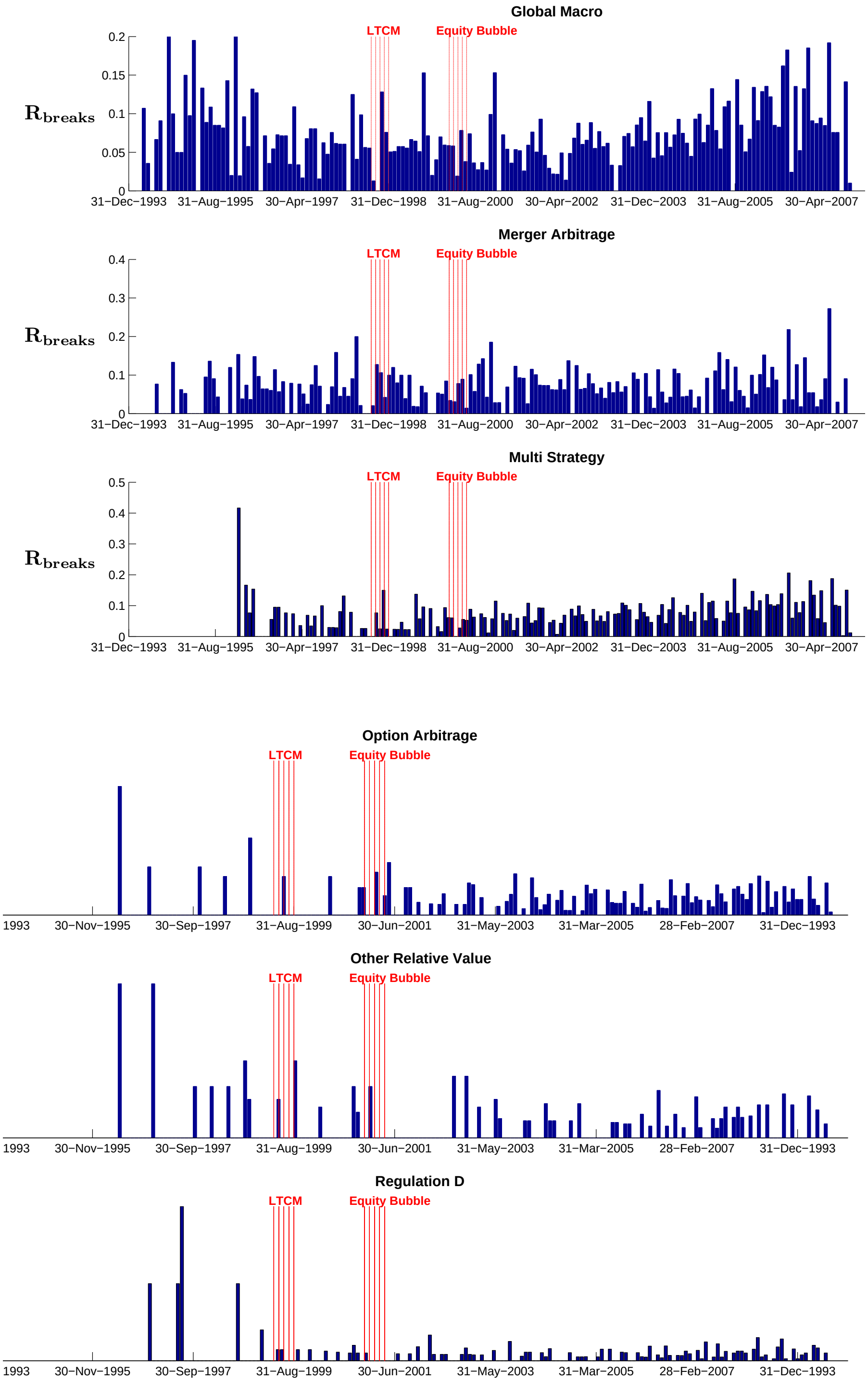
See Appendix I for definitions of fund types. The bar figures illustrate the percentage of breakdates relative to the Hedge Fund population which were in activity. The red bar cover the two studied period i.e. LTCM and the equity Bubble.

Figure II: Structural breaks over the period January 1994 - April 2007



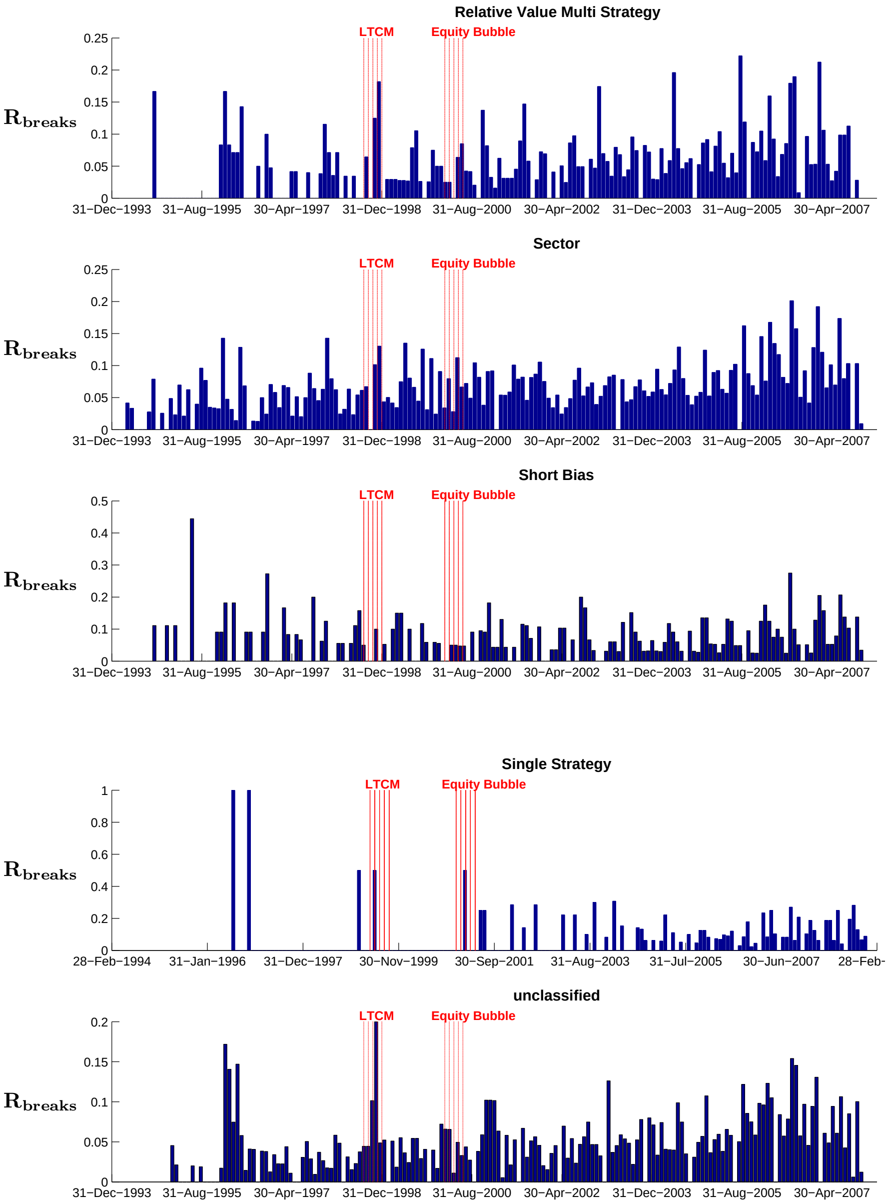
See Appendix I for definitions of fund types. The bar figures illustrate the percentage of breakdates relative to the Hedge Fund population which were in activity. The red bar cover the two studied period i.e. LTCM and the equity Bubble.

Figure III: Structural breaks over the period January 1994 - April 2007



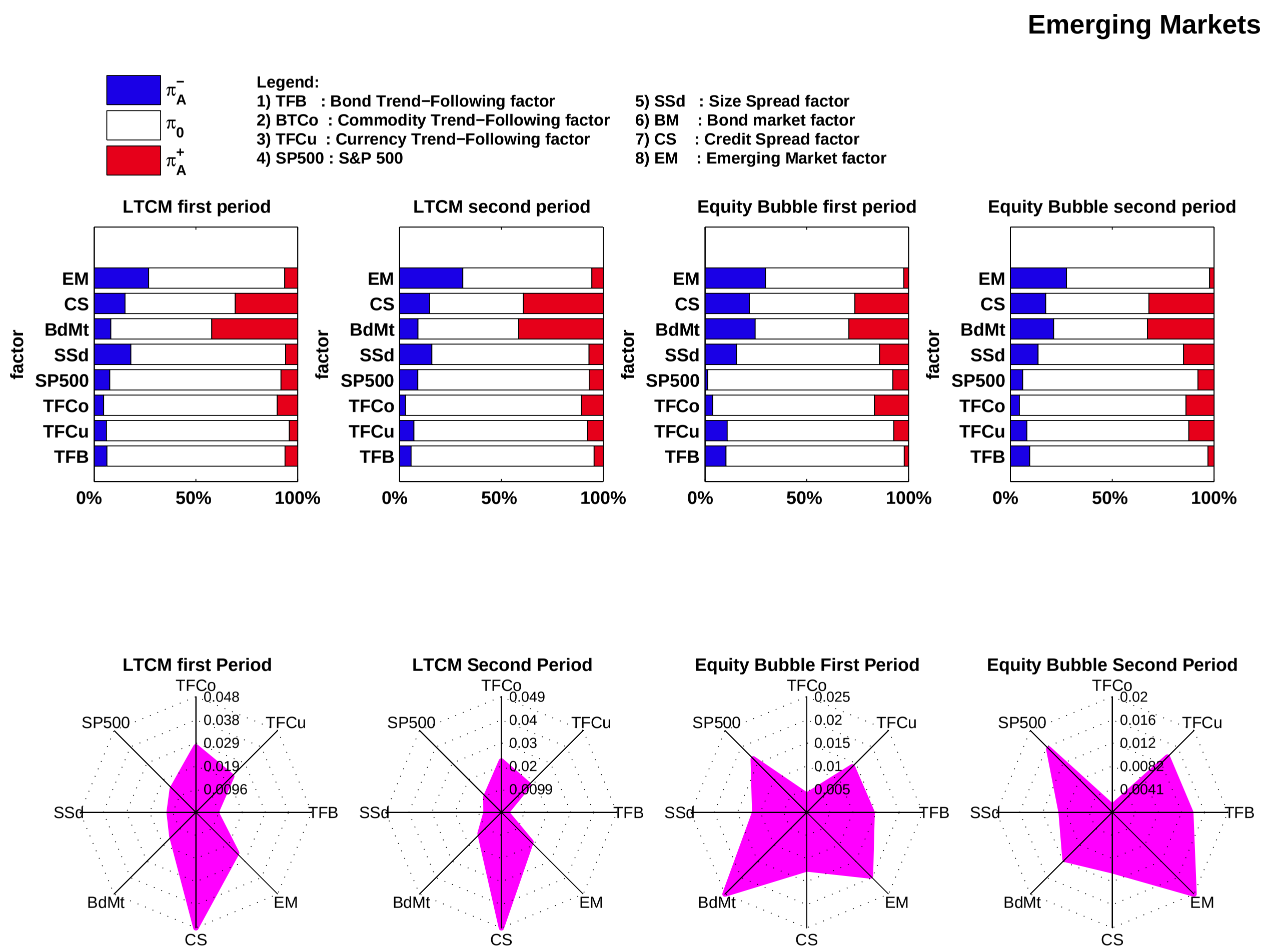
See Appendix I for definitions of fund types. The bar figures illustrate the percentage of breakdates relative to the Hedge Fund population which were in activity. The red bar cover the two studied period i.e. LTCM and the equity Bubble.

Figure IV: Structural breaks over the period January 1994 - April 2007



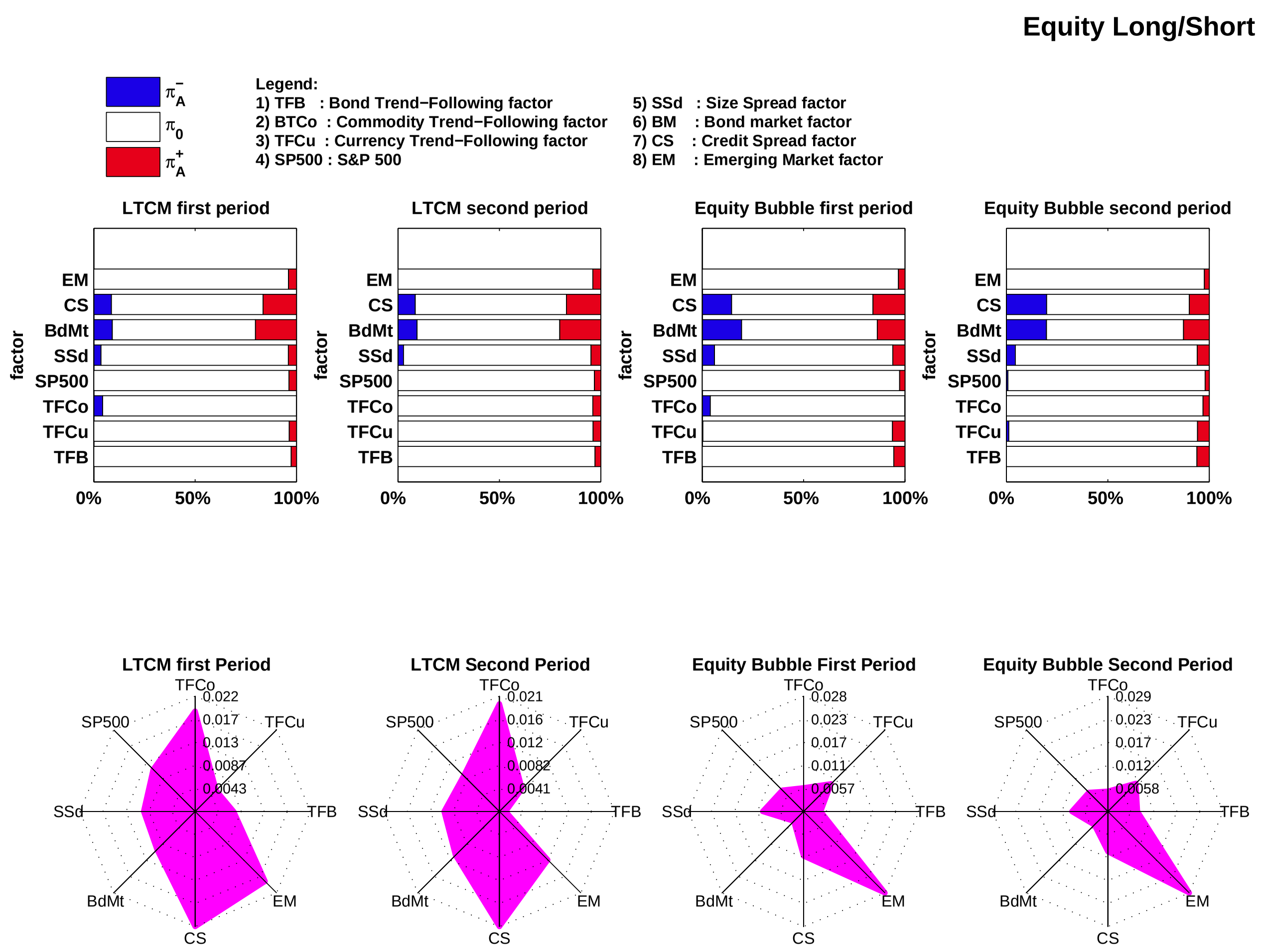
See Appendix I for definitions of fund types. The bar figures illustrate the percentage of breakdates relative to the Hedge Fund population which were in activity. The red bar cover the two studied period i.e. LTCM and the equity Bubble.

Figure V: Variation in Market Exposures during Turmoils: Emerging Market



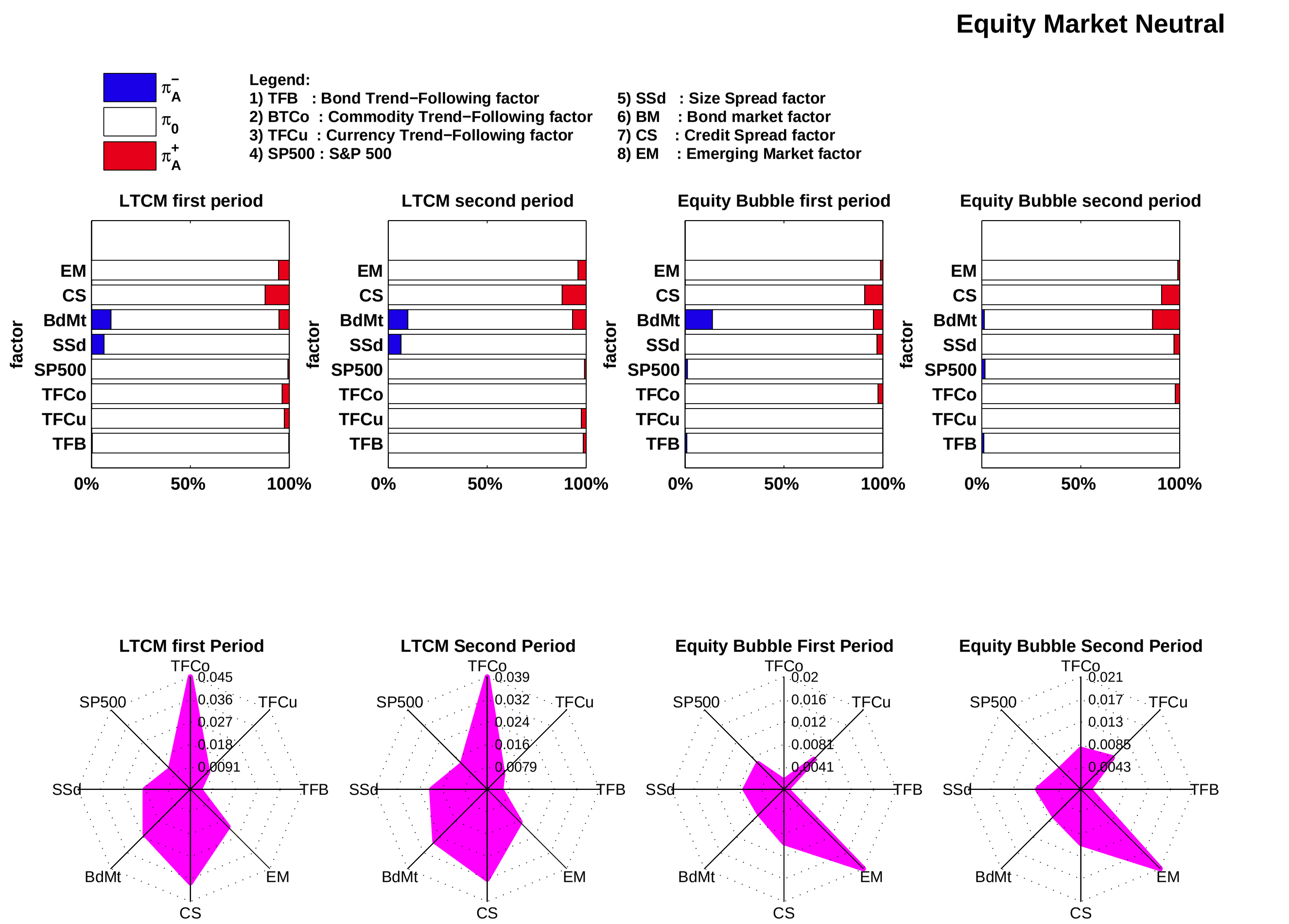
See Appendix I for definitions of fund types. The number of funds covering the period is equal to 461. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Figure VI: Variation in Market Exposures during Turmoils: Equity Long/Short



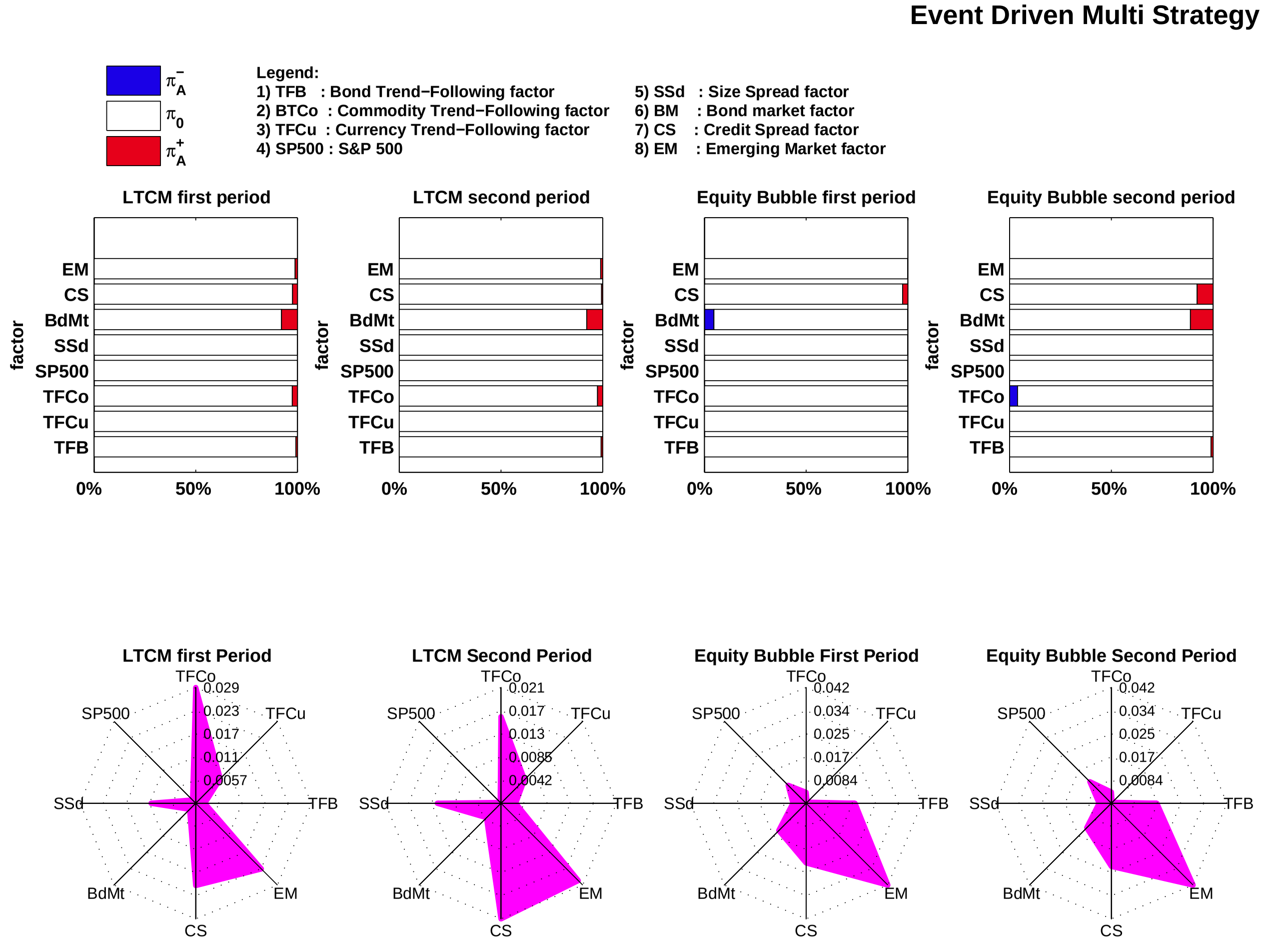
See Appendix I for definitions of fund types. The number of funds covering the period is equal to 2525. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Figure VII: Variation in Market Exposures during Turmoils: Equity Market Neutral



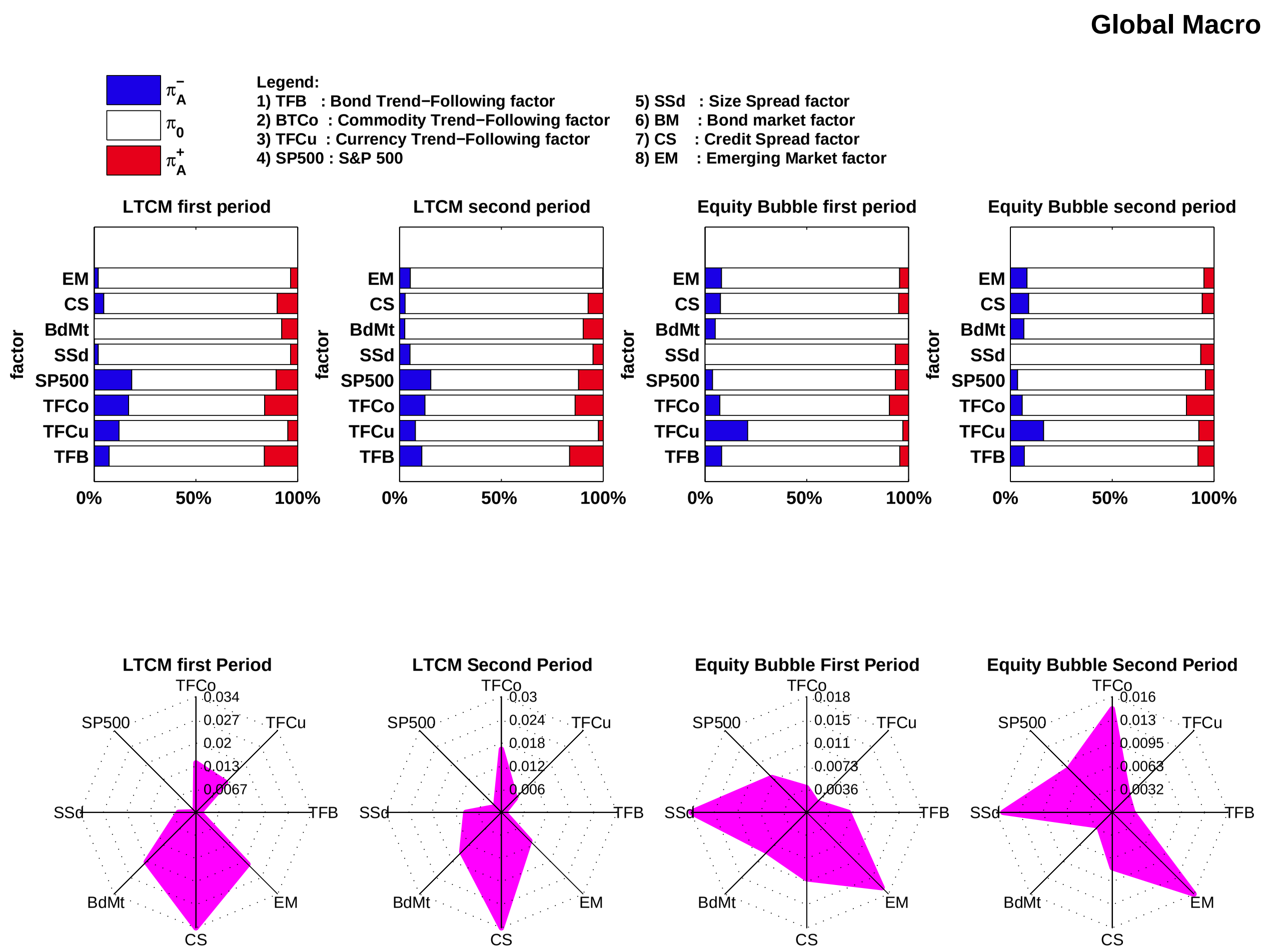
See Appendix I for definitions of fund types. The number of funds covering the period is equal to 519. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Figure VIII: Variation in Market Exposures during Turmoils: Event Driven Multi Strategy



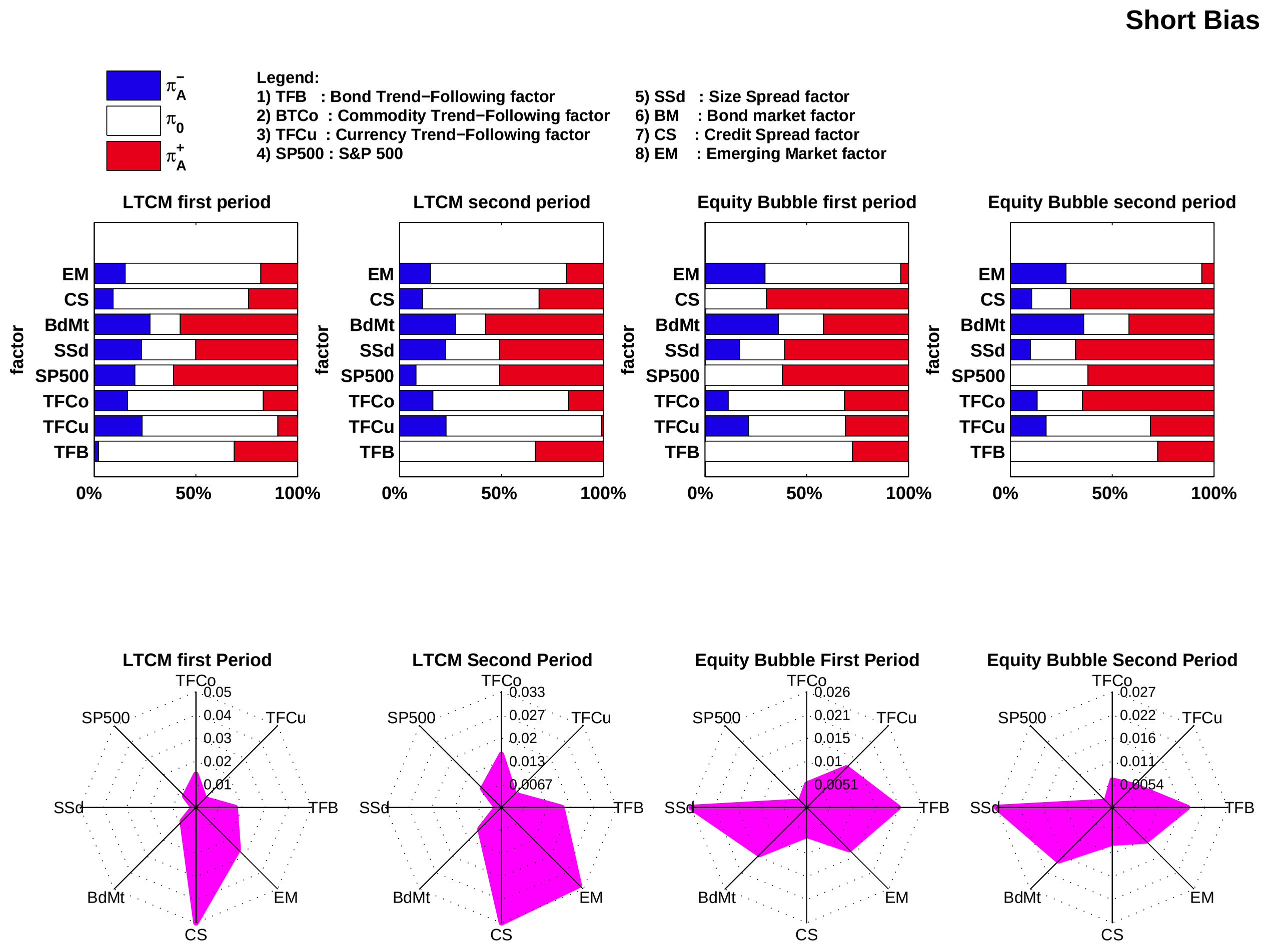
See Appendix I for definitions of fund types. The number of funds covering the period is equal to 293. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Figure IX: Variation in Market Exposures during Turmoils: Global Macro



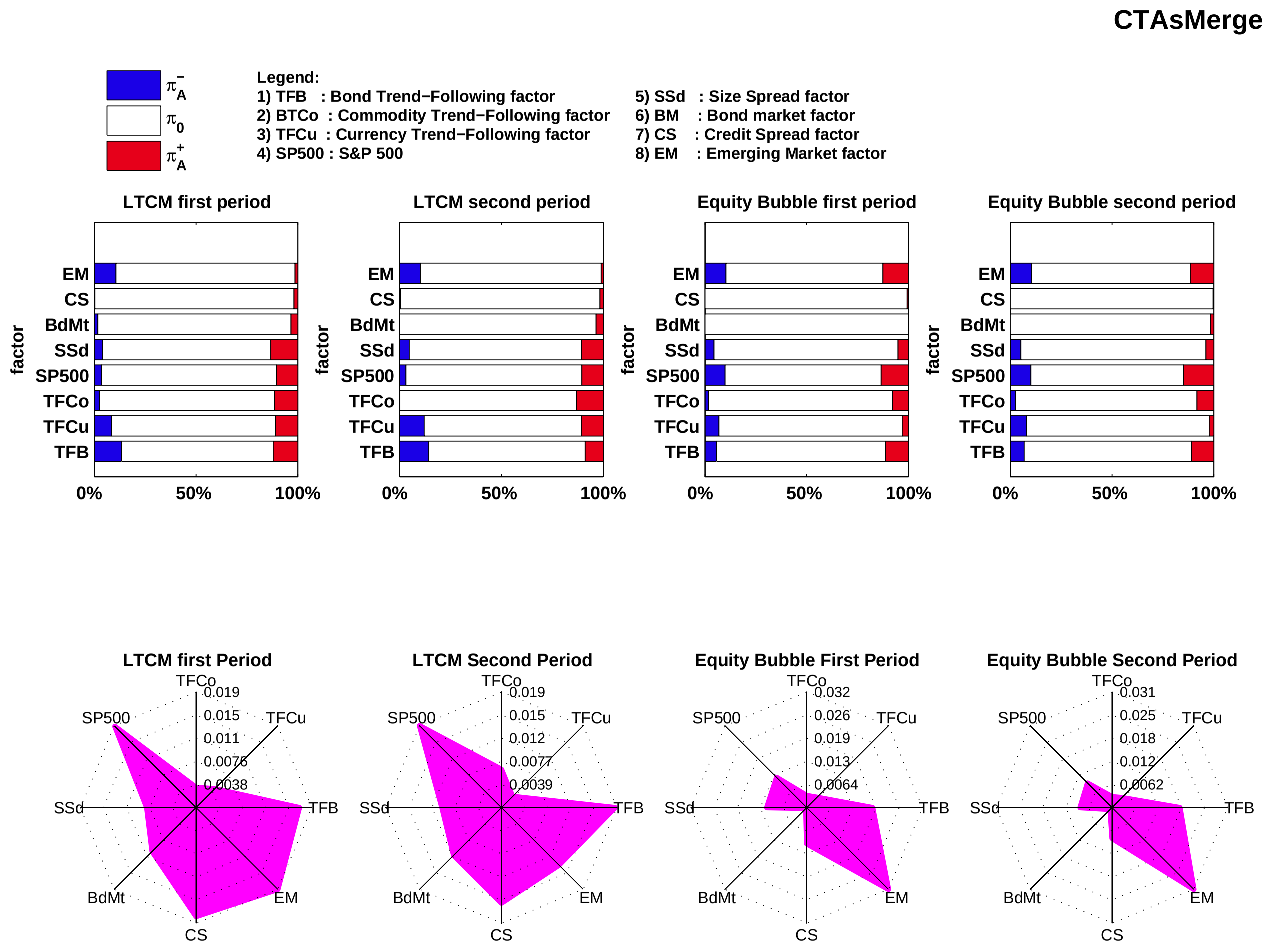
See Appendix I for definitions of fund types. The number of funds covering the period is equal to 377. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Figure X: Variation in Market Exposures during Turmoils: Short Bias



See Appendix I for definitions of fund types. The number of funds covering the period is equal to 56. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Figure XI: Variation in Market Exposures during Turmoils: CTA



See Appendix I for definitions of fund types. The number of funds covering the period is equal to 1394. The bar figures illustrate the proportion of funds having more than 10 percent decrease (blue or π_A^-), constant (white or π_0), and increase (red or π_A^+) market exposure during the 2 crises. Each crisis is divided in 2 period (see Section 7: methodology). . The second figures (Factor Radar Chart) indicates the strategy's sensitivities (percentage change) to various factors in regards to the 2 crises.

Table VII: Proportion of Unskilled and Skilled Funds during the Subprime Mortgage Crisis

Performance is measured with our time-varying coefficient model over the period 31/03/2007 to 31/03/2010. The table displays the estimated proportions of zero-alpha, unskilled, and skilled funds (π_0, π_A^-, π_A^+) for each strategy.

Strategy	number of funds	π_A^+	π_0	π_A^-
Equity Long/Short	2000	12.42%	12.32%	75.26%
Convertible Arbitrage	123	21.6%	23.3%	55.1%
CTA	1096	24.6%	28.3%	47%
Event Driven Multi Strategy	439	27.5%	22.5%	50.%
Emerging Market	342	21.88%	26.92%	51.2%
Fixed Income	507	25.3%	19.3%	55.4%
Global Macro	813	22%	22.6%	55.4%
Short Selling	28	34.4%	25%	40.6%
Equity Market Neutral	436	27%	27.29%	49.61%

Appendix I

Definition of Strategies

- The **emerging markets strategy** attempts to capture gains from inefficiencies in emerging markets.
- The **Equity Long/Short strategy** refers to taking both long and short positions in equities.
- The **Market Timer** focus on securities associated with companies that will soon experience a significant event.

  The **Distressed Securities Strategy** focuses on asset of distressed companies. Buys equity, debt, or trade claims at deep discounts of companies in or facing bankruptcy or reorganization.

- The **Merger Arbitrage Strategy** also called **risk arbitrage strategy** exploit pricing inefficiencies associated with a merger or acquisition.
- The **event driven multi-strategy** can use both the distressed securities style and/or the merger arbitrage style.
- The **Relative Value arbitrage style** take positions in 2 securities that are mispriced relative to each other, with the expectation that their prices will converge to appropriate values in the future(Arbitrage, Market neutral.
- The **arbitrage** involves simultaneously purchasing and selling related securities that are mispriced relative to each other.
- **Convertible Arbitrage Strategy** can be described by taking a long position in a convertible bond and sells short the associated stock. Convertible arbitrage - exploit pricing inefficiencies between convertible securities and the corresponding stocks.

- **Fixed Income Arbitrage Strategies** encompass a wide range of strategies in both domestic and global fixed-income markets. Fixed income arbitrage - exploit pricing inefficiencies between related fixed income securities.
- **Equity Market Neutral style** creates a position that attempts to hedge out most market risk by taking offsetting positions. This strategy exploits the mispricing between a stock which is overvalued and one that is undervalued such that beta of the combined position is zero. Statistical arbitrage - equity market neutral strategy using statistical models.
- **Index arbitrage style** generally attempts to exploit mispricing between an index and derivatives on that index.
- **Mortgage-backed securities arbitrage style** exploit the mispricing of mortgage-backed assets relative to Treasury securities.
- **multi-strategy style** uses different styles and may change exposures to different styles based upon changing market conditions.Multi strategy in Macro strategy - combination of discretionary and systematic macro. Multi strategy in FoHF - a hedge fund exploiting a combination of different hedge fund strategies to reduce market risk.
- **dedicated short selling style** only takes short equity positions.
- **Global Macro** Discretionary macro - trading is done by investment managers instead of generated by software.
  Systematic macro (Systematic diversified) - trading is done mathematically, generated by software without human intervention.
- **Sector funds** - expertise in niche areas such as technology, health care, biotechnology, pharmaceuticals, energy, basic materials.
- **Fundamental value** - invest in undervalued companies.
- **Fundamental growth** - invest in companies with more earnings growth than the broad equity market.
- **Quantitative Directional, statistical arbitrage** - equity trading using quantitative techniques.
- **Multi manager** - a hedge fund where the investment is spread along separate sub managers investing in their own strategy.
- **Trend following** - long-term or short-term. Non-trend following (Counter trend) - profit from trend reversals.
- **Regulation D** - specialized in private equities.
- **Credit arbitrage or fixed income arbitrage strategy** - specialized in corporate fixed income securities.
- **Fixed Income asset backed** - fixed income arbitrage strategy using asset-backed securities.
- **Volatility arbitrage** - exploit the change in implied volatility instead of the change in price.
- **Yield alternatives** - non fixed income arbitrage strategies based on the yield instead of the price.
- **Capital Structure Arbitrage** - involves taking long and short positions in different financial instruments of a company's capital structure, particularly between a company's debt and equity product.

Two-step Time-Varying Coefficient Model

The varying coefficient model assumes the following conditional linear structure:

$$Y_t = \sum_{j=1}^{p} \beta_j(t) X_{jt} + \varepsilon_t = \alpha(t) + X\beta(t) + \varepsilon_t$$

for a given covariates $(t, X_1, ..., X_p)'$ and variable Y with

$$E[\varepsilon|t, X_1, ..., X_p] = 0,$$

$$Var[\varepsilon|t, X_1, ..., X_p] = \sigma^2(t),$$

In this study, we took $X_1 = 1$ as the intercept term and $t = time$.

if we consider that β_i depends on t: $(\beta_i(t))$, we can approximate the function locally as $\beta_i(t) \approx a_i + b_i(t - t_0)$. This leads to the following local least-squares problem:

$$minimize \quad \sum_{i=1}^{n} \left[Y_i - \sum_{j=1}^{p} \{a_j + b_j(T_i - t_0)\} X_{ij} \right]^2 K_h(T_i - t_0),$$

for a given kernel function K and bandwidth h, where $K_h(.) = K(./h)/h$.
In matrix notation:

Let

$$\mathbf{X}_0 = \begin{pmatrix} X_{11} & X_{11}(T_1 - t_0) & \dots & X_{1p} & X_{1p}(T_1 - t_0) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ X_{n1} & X_{n1}(T_n - t_0) & \dots & X_{np} & X_{np}(T_p - t_0) \end{pmatrix},$$

$$Y = (Y_1, ..., Y_n)' \; and \; W_0 = diag(K_{h_0}(T_1 - t_0), ..., K_{h_0}(T_n - t_0)).$$

Then the solution to the least-squares problem can be expressed as:

$$\widehat{a}_{j,0} = e'_{2j-1,2p}(\mathbf{X}'_0 W_0 \mathbf{X}_0)^{-1} \mathbf{X}'_0 W_0 Y.$$

With these estimates, $\widehat{a}_{j,0}$, a local least-square regression is fitted again via substituting the initial estimate into the local least-squares problem:

$$\sum_{i=1}^{n} \left(Y_i - \sum_{j=1}^{p-1} \hat{a}_{j,0}(T_i) X_{ij} - \left\{ a_p + b_p(T_i - t_0) + c_p(T_i - t_0)^2 + d_p(T_i - t_0)^3 \right\} X_{ip} \right)^2 \times K_{h_2}(T_i - t_0),$$

where h_2 is the bandwidth in the second step. In this way, a two-step estimator is obtained. Fan and Zhang showed that the bias of the two-step estimator is of $\mathcal{O}(h_2^4)$ and the variance $\mathcal{O}\left\{(nh_2)^{-1}\right\}$

Two-step Time-Varying Coefficient Model using B-splines

As mentioned by Fan and Zhang (1999), other techniques such as smoothing splines can also be used in the second stage of fitting. Therefore we built the second two-step estimator based on the same article but we used a smoothing splines instead of local regression during the second step.
From the first step, we obtained the estimates:

$$\widehat{a}_{j,0} = e'_{2j-1,2p}(\mathbf{X}'_0 W_0 \mathbf{X}_0)^{-1}\mathbf{X}'_0 W_0 Y,$$

In a second step, knowing that we can approximate each $\beta_p(t)$ by a basis function expansion

$$\beta_p(t) \simeq \sum_{k=0}^{K} \gamma^*_{pk} B_{pk}(t).$$

We can now minimize in order to estimate γ^*_{kp}:

$$\sum_{i=1}^{n} w_i \left(Y_i - \sum_{j=1}^{p-1} \hat{a}_{j,0}(T_i) X_{ij} - \left\{ \sum_{k=0}^{K} \gamma_{kp} B_{kp} \right\} X_{ip} \right)^2,$$

and we estimate $\beta_p(t)$ by $\hat{\beta}_p(t) = \sum_{k=1}^{K} \hat{\gamma}_{kp} B_{kp}(t)$.

Monte Carlo Analysis: local and B-spline Time-Varying Coefficient Model

The goal of this section is to demonstrate that our model succeeds in capturing structural change. For this, we create a track of return where the two betas have a structural change. We allow for three different sample sizes: fifty, one hundred and one hundred and fifty months. These akin to the sizes that we can find in different databases.

In order to show that our results are independent to our estimation methodology. In second step to our estimator we used a B-spline smoothing instead of a local regression. We retest the ability to capture a structural break. The results are also very good.
We use one of the same example as in Zhang, Lee and Song (2002).We apply the false discovery rate to the estimated alpha and betas obtained by B-spline. The results give approximately the same percentage of unskilled, zero and skilled funds for our population.

The following example will be used to illustrate the performance of our estimator. We created two “betas ”(called $\beta^i_{created}$) which represent a possible structural change for a Hedge Fund. We design a simulated Hedge Fund track (R_{HF}) in this manner:

$$R_{HF} = \beta^1_{created}X_1 + \beta^2_{created}X_2 + \varepsilon$$

where X_1, X_2 are the S&P500 and the monthly change in the 10-year treasury constant maturity yield respectively. The random variable ε follows a normal distribution with mean zero and variance 1.
We called the local time-varying coefficient model: L-TVCM and the B-spline time varying coefficient model: B-TVCM.

Short sample: 50 months

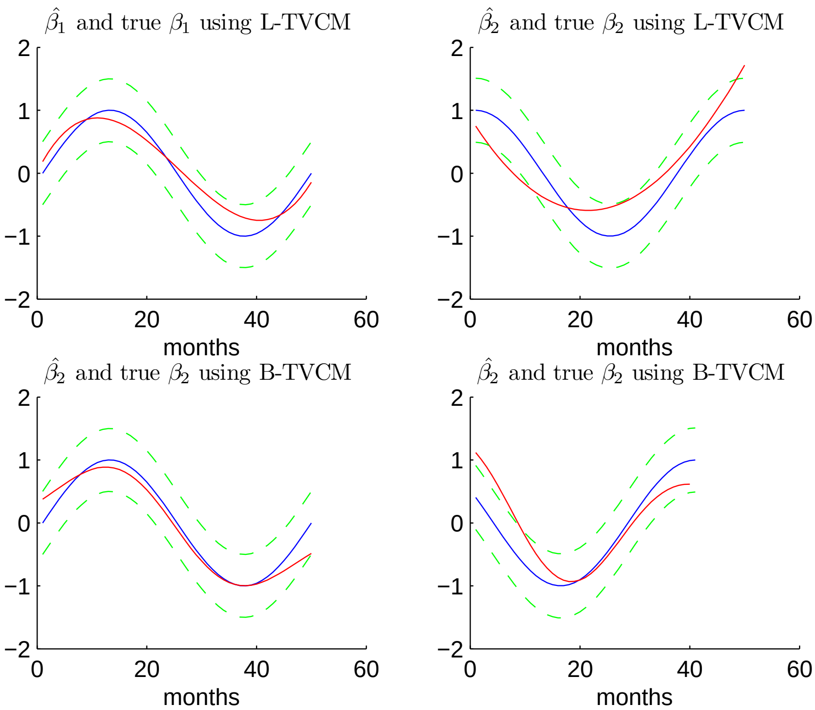


FIG.: Comparison of the performance between the one-step estimator (long-dashed curve) and the true coefficient function (the solid curve).

Medium sample: 100 months

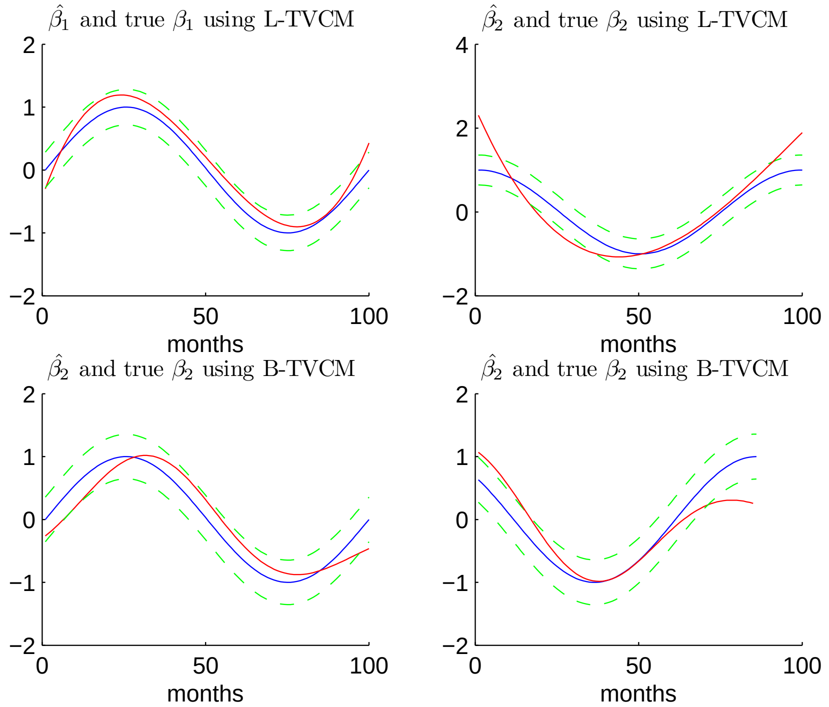


FIG.: Comparison of the performance between the one-step estimator (long-dashed curve) and the true coefficient function (the solid curve).

Long sample: 150 months

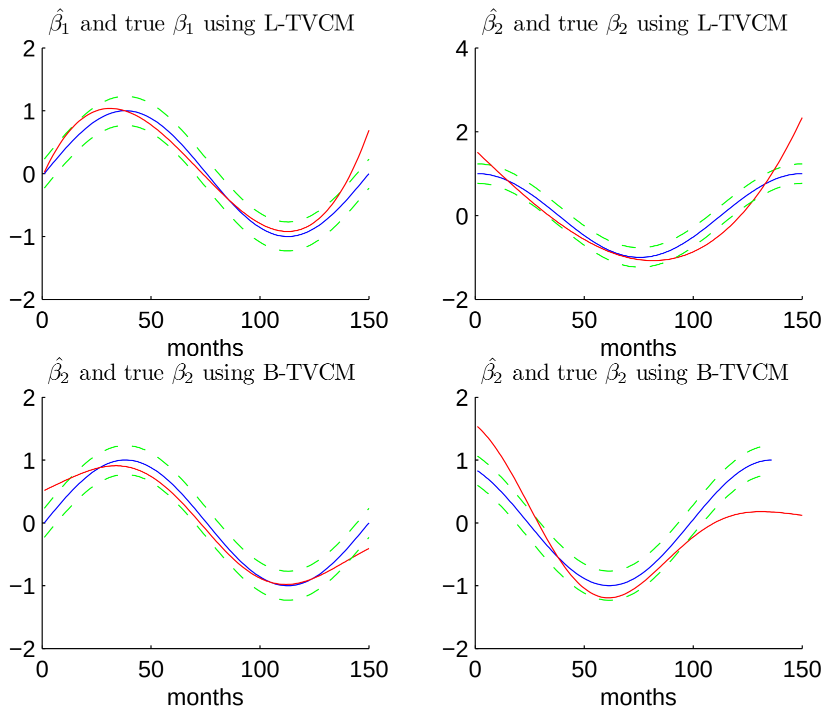


FIG.: Comparison of the performance between the one-step estimator (long-dashed curve) and the true coefficient function (the solid curve).

Bootstrap Confidence intervals

We summarize the methodology from Colin and Chiang (2000) in order to create confidence regions.

According to the author this following naive bootstrap procedure can be used to construct approximate pointwise percentile confidence intervals for $\beta_i(t)$:

- 1) Randomly sample n subjects with replacement from the original data set, and let $\{(t^*_{ij}, \mathbf{X}^*_i), Y^*_{ij}; 1 \neq i \neq n, 1 \neq j \neq n_i\}$ be the longitudinal bootstrap sample.
- 2) Compute the kernel estimator $\widetilde{\beta}_i^{boot}(t)$
- 3) Repeat the above 2 steps B times, so that B bootstrap estimators $\widetilde{\beta}_i^{boot}(t)$ of $\beta_i(t)$ are obtained.
- 4) Let $L_{\alpha/2}(t)$ and $U_{(\alpha/2)}(t)$ be the $(\alpha/2)^{th}$ and $(1-\alpha)^{th}$ i.e. lower and upper $(\alpha/2^{th})$ percentiles, respectively, calculated on the B bootstrap estimators. An approximate $(1-\alpha)$ bootstrap confidence interval for $\beta_i(t)$ is given by $(L_{(\alpha/2)}(t), U_{(\alpha/2)}(t))$.

Bandwidth "leave-one-subject-out" cross-validation methodology

The leave-out method is based on regression smoothers in which one, say the jth, observation is left out. So, for N values,

- 1) Compute the leave-out estimate $\hat{m}_{h,j}(X_j) = n^{-1} \sum\limits_{i \neq j} W_{hi}(X_j) Y_i$.
- 2) Construct the cross validation function $CV(h) = n^{-1} \sum\limits_{j=1}^{n} (Y_j - \hat{m}_{h,j}(X_j))^2 w(X_j)$. where w denotes a weight function.
- 3) With this N CV(h), we can, now, define the automatic bandwidth as $\hat{h} = arg \min\limits_{h \in H_n} [CV(h)]$

Blundell-ward filter

The observed (or smoothed) value V^*_t of a Hedge Fund at time t could be expressed as a weighted average of the true value at time t, V_t and the smoothed value at time $t-1$, V^*_{t-1}:

$$V^*_t = \alpha V_t + (1-\alpha) V^*_{t-1},$$

$$r_t = \frac{r^*_t - \alpha r^*_{t-1}}{1-\alpha}.$$

.